Subjective Performance Evaluation of Employees with Biased Beliefs

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Abstract

This paper analyzes how worker optimism (and pessimism) affects subjective performance evaluation (SPE) contracts. An optimistic (pessimistic) worker overestimates (underestimates) the probability of observing an acceptable performance. The firm is better informed about performance than the worker and knows the worker's bias. We show that optimism (and pessimism) can: (i) change the optimal incentive scheme under SPE, (ii) lower the deadweight loss associated with SPE contracts, (iii) lead to a Pareto improvement by simultaneously lowering the firm's expected wage cost and raising the worker's expected compensation. In addition, we show that worker pessimism can lead to SPE contracts without a deadweight loss.

1. Introduction

Most workers perform jobs where objective performance measures are extremely difficult to obtain (Prendergast, 1999). Very often the ultimate quality of a worker's performance, output or service is not directly observable. This happens in the production of complex goods like movies, technological gadgets, or academic research papers. In these types of jobs firms tipically use subjective performance measures to provide work incentives. For example, subjective evaluations of supervisors, co-workers, or consumers.

The absence of objective performance measures creates a natural environment for biases like overconfidence and optimism to influence economic behavior.[1] Optimism is a well documented psychological phenomenon. Most individuals tend to overestimate their chances of experiencing positive and underestimate their chances of experiencing negative events (e.g. Weinstein, 1980; Taylor and Brown, 1988). Arabsheibani,

[1]Felson (1981) and Dunning, Meyerowitz, and Holzberg (1989) show empirically that the more ambiguous or subjective is the definition of an ability, the more individuals overestimate of their relative skills (a form of overconfidence). Van Den Steen (2004) and Santos-Pinto and Sobel (2005) provide mechanisms whereby an increase in subjectivity raises optimism and overconfidence.

De Meza, Maloney, and Pearson (2000) find that entrepreneurs, managers, and workers are optimistic about their financial outcomes. Koudstaal, Sloof, and Van Praag (2015) find that entrepreneurs, managers, and workers display dispositional optimism, i.e., the global expectation that good things will be plentiful in the future and bad things will be scarce (e.g. Scheier and Carver, 1985; Scheier, Carver, and Bridges, 1994; Peterson, 2000). Optimism matters for economic decisions like market entry, portfolio, and career choices (e.g. Puri and Robinson, 2007). While optimistic biases are a robust and widespread psychological phenomenon, pessimistic biases are rare. Still, some individuals tend to underestimate their chances of experiencing positive events and overestimate their chances of experiencing negative ones. Carver, Scheier, and Segerstrom (2010) review the literature on optimism and show how "it is [...] possible to identify people who are pessimists in an absolute sense" and that "doing this reveals that pessimists are a minority".

In this paper we ask how does worker optimism and pessimism affect the optimal design of subjective performance evaluation contracts? Is there a way for the employer to take advantage of the bias of the worker? Does the worker lose or gain from being optimistic/pessimistic? How does the presence of biased workers affect social welfare? In the following sections we provide precise answers to these questions and show that the SPE contracts offered to biased workers may differ substantially (qualitatively and quantitatively) from the ones offered to unbiased ones. We find that the features of these contracts lead to three main welfare results. First, the principal can take advantage of the bias of the worker in order to decrease the cost of implementing high effort. That is, the principal is always (at least weakly) better off when the worker is optimistic/pessimistic, compared to the case of an unbiased worker. Second, optimism and pessimism can lower the deadweight loss of subjective performance evaluation contracts.[2] Under some specific conditions, the misalignment of beliefs between the principal and the agent may even lead to contracts that feature no deadweight loss at all. Third, workers' biases can lead to a Pareto improvement by simultaneously lowering the firm's expected wage cost and raising the worker's expected compensation.

In our model, we consider a contractual environment where a risk neutral firm (or principal) offers a one period contract to a risk neutral worker (or agent). If the agent accepts the contract he chooses an effort level: high or low. The probability that a benefit is realized is larger under high effort than under low effort and the cost of exerting high effort is larger than the cost of exerting low effort. The effort choice of the agent as well as the benefit are not directly observable. However, the benefit generates separate private (and hence subjective) signals for the principal and the agent. We assume that each signal only has two possible realizations: acceptable and unacceptable performance. The signals are imperfectly positively correlated and

[2]Deadweight loss is a standard feature of SPE contracts (see the seminal paper MacLeod (2003) and the literature we cite below) and is often referred to also as "money burning".

the extent to which performance evaluations are subjective depends on the degree of correlation of the signals. The principal's signal is more informative than that of the agent. We focus on the case where the principal decides to implement high effort. A contract in our set-up specifies a wage cost for the principal and a compensation for the agent under each reported state. The wage is the principal's dollar cost of employing the agent and the compensation is the dollar amount the agent receives. We allow the agent to costlessly impose a deadweight loss upon the principal. This captures the notion of conflict in a relationship which might happen when the parties disagree on their performance evaluations. In the examples cited above, for instance, the worker may decide to "punish" the employer by performing badly, signing for another team, changing manager and so on.[3]

In this framework, an optimistic (pessimistic) agent overestimates (underestimates) the probability of observing an acceptable performance given the realization of the principal's signal. The principal is fully informed about the bias of the agent.

While optimism and pessimism already separate types of agents into two categories, the fact that the bias of the agent applies only to his own signal and not to the one of the principal also affects the perceived correlation of the signals. We distinguish between agents who perceive signals to be positively correlated (as they actually are) and agents who perceive (mistakenly) signals to be negatively correlated. This proves to be a key distinction since the classical result on the existence of conflict and deadweight loss in SPE contracts does not necessarily hold any longer for the case of negative perceived correlation. When the principal and the agent disagree only about the degree of positive correlation between the signals, the principal can "speculate" on the compensation granted to the agent by promising more (less) in states the agent deems more (less) probable than the principal does. This alleviates the conflict present in the contract and may lead to Pareto improvements compared to the case of an unbiased agent. It is not, however, enough to rule out conflict entirely. When the perceived correlations differ also in direction, however, not only the principal can speculate on the states mentioned, but now the states deemed most probable by the two parties are exact opposites. This creates an incentive for the principal to speculate even further. In face of a greater (believed) expected compensation, the agent may find it optimal to sign a contract that features no conflict and no deadweight loss.

The rest of the paper is organized as follows. Section 2 discusses the related literature and compares it to our findings. Section 3 sets-up the model, shows how we introduce and model workers' biases in the model, formalizes the principal's effort implementation

[3]Mas (2006, 2008) provides direct evidence of employees imposing direct costs upon employers through private actions. These costs include a decrease in future effort (Mas, 2006) or a direct reduction in the quality of the output (Mas, 2008).However, there are also examples of employers imposing direct costs upon employees. In the sports and entertainment businesses, athletes and performers (e.g., actors and musicians) are often subject to fines or are not called up for a particular game or show.

problem, and states some basic features of optimal contracts in our set-up. Section 4 solves the model in the presence of an optimistic agent while section 5 assumes a pessimistic agent. Section 6 solves the model in the presence of two further types of biased agents allowed by the model. We name them "trusty" and "skeptical". Section 7 presents a thorough welfare analysis of each of the new contracts and discusses the main results on welfare and social value of workers' biases. Section 8 concludes the paper. All proofs are relegated to the appendix.

2. Related Literature

Our paper contributes to the literature on subjective performance evaluation. Within this literature the closest paper to ours is MacLeod (2003). He shows that when signals are perfectly correlated the incentive constraints for the revelation of subjective information are not binding and the optimal contract with subjective evaluation is the same as the optimal principal-agent contract with verifiable information. In this case there is no welfare loss due to the incentive constraints arising from subjective evaluation. This is no longer the case when signals are imperfectly correlated. MacLeod (2003) also shows that the agent's ability to harm the principal can be an essential input into an optimal contract with subjective evaluation. Furthermore, MacLeod (2003) shows that a higher level of correlation between the parties' information reduces the expected level of conflicts in an optimal contract.

Our paper also contributes to the growing literature on the impact of biased beliefs on the employment relationship.[4] In accordance with our results, this literature highlights further cases where workers' optimism or overconfidence may have positive welfare implications. Hvide (2002) shows that worker overconfidence about productivity outside the firm improves worker welfare. Bénabou and Tirole (2003) show that if a firm is better informed about a worker's skill than the worker, effort and overconfidence are complements, then the firm has an incentive to boost the worker's overconfidence by offering low-powered incentives that signal trust to the worker and increase motivation. Gervais and Goldstein (2007) find that a firm is better off with a team of workers who overestimate their skill when there are complementarities between workers' efforts. Further, in this literature are a set of papers that, like ours, study the implications of the presence of a biased agents on key contractual aspects. Santos-Pinto (2008, 2010) and De la Rosa (2011) show how firms can design objective performance evaluation contracts to take advantage of worker overconfidence about productivity inside the firm. Fang and Moscarini (2005) and Santos-Pinto (2012) show that worker overconfidence can lead to wage compression inside and outside the firm, respectively.

Finally, our paper contributes to the literature on exploitative contracting. Section 7 provides conditions under which the new contracts we derive do not necessarily feature

[4]We will focus on papers that use the principal-agent framework to model the worker-firm relationship.

an "exploitative nature" in the sense of making the principal better off and the agent worse off (compared to the case of an unbiased agent). When these conditions aren't met, however, the principal does *exploit* the agent's biased beliefs. Notable and related contributions are Della Vigna and Malmendier (2004), Gabaix and Laibson (2006), Eliaz and Spiegler (2008), Heidhues and Koszegi (2010), and Foschi (2017). Della Vigna and Malmendier (2004) show how firms can design contracts to take advantage of consumers with quasi-hyperbolic preferences. Gabaix and Laibson (2006) show how firms can use base-good and add-on pricing schemes to exploit consumers who are unaware of the existence of the add-on. Eliaz and Spiegler (2008) study optimal dynamic contracting when agents are uncertain about their own preferences (naïve agents) at the time of signing the contract, and they may be more optimistic than the principal about the better state occurring. Heidhues and Koszegi (2010) study exploitative credit contracts. Foschi (2017) studies the design of optimal contracts for naïve agents introducing the assumption that naïveté may depend on the ability agents are uncertain about.

3. A Binary Model of Subjective Evaluation

In this section we set-up the model, define an optimistic (pessimistic) agent, formalize the principal's problem, and describe some basic features of optimal contracts in our set-up.

3.1. **Set-up.** A risk neutral principal (she) offers a one period contract to an agent (he). If the agent accepts, he chooses effort $\lambda \in \{\lambda^L, \lambda^H\}$, where λ is the probability that output Y is realized. We let both λs belong to $[0,1]$ with $\lambda^H > \lambda^L$. The net benefit to the principal is:

$$E(\Pi) = \lambda Y - E(w)$$

where w are the dollar costs of employing the agent and return Y is always strictly positive. We say that the result of the project is "good" ("bad") if Y is (is not) realized.

When the agent exerts effort λ, he obtains $U(c,\lambda) = u(c) - V(\lambda)$, where c is the compensation for his work and $V(\lambda)$ is the cost of the effort exerted (with $V(\lambda^H) > V(\lambda^L) \geq 0$). In this paper we derive the optimal contract when the principal faces a risk neutral agent and there is limited liability. Hence, we assume $u(c) = c$ and $c \geq 0$. We also assume that the agent has access to an outside option granting him $\overline{u}$.

Following MacLeod (2003), neither the outcome of the project nor the effort exerted are observable. The outcome of the project generates separate private (and hence subjective) signals for the principal and the agent. The principal observes a measure of performance or signal T and the agent observes a measure of performance or signal S. Signal T has a realization $t \in \{a,u\}$ and signal S has a realization $s \in \{a,u\}$.

Realization a (u) corresponds to an "acceptable" ("unacceptable") performance.[5] In particular, we let $\gamma_t^G = \Pr\{T = t|G\}$ and $\gamma_t^B = \Pr\{T = t|B\}$ be the probability that signal T results in $t \in \{a, u\}$ when the outcome of the project is good (G) or bad (B) respectively.

Assumption 1. *Signal T is positively correlated with the outcome of the project. That is:*

$$\gamma_a^G > \gamma_a^B \quad \text{and} \quad \gamma_u^G < \gamma_u^B$$

The realization of signal S is described as a function of T. Let $P_{ts} = \Pr\{S = s|T = t\}$. The unconditional probability of realizations ts to occur together in state G and B are $\gamma_{ts}^G = \Pr\{T = t, S = s|G\} = P_{ts}\gamma_t^G$ and $\gamma_{ts}^B = \Pr\{T = t, S = s|B\} = P_{ts}\gamma_t^B$. This allows us to derive the following probabilities, crucial for contracting:

$$\gamma_{ts}^H = \Pr\{T = t, S = s|\lambda^H\} = \lambda^H\gamma_{ts}^G + (1 - \lambda^H)\gamma_{ts}^B$$
$$\gamma_{ts}^L = \Pr\{T = t, S = s|\lambda^L\} = \lambda^L\gamma_{ts}^G + (1 - \lambda^L)\gamma_{ts}^B$$

Assumption 2. *For any λ^j, signals are positively correlated in the following sense:*

$$\gamma_{aa}^j\gamma_{uu}^j - \gamma_{au}^j\gamma_{ua}^j > 0$$

Assumption 2 has implications also on the conditional distributions of signals.

Lemma 1. *Given the positive correlation of signals the following are true:*

(i) $\gamma_{ts}^j = P_{ts}\Pr[T = t|\lambda^j] \equiv P_{ts}\Gamma_t^j$,
(ii) $P_{aa}P_{uu} - P_{au}P_{ua} > 0$,
(iii) $P_{aa} > P_{ua}$ *and* $P_{uu} > P_{au}$.
(iv) $\Delta\Gamma_a + \Delta\Gamma_u = 0$, *where* $\Delta\Gamma_t = \Gamma_t^H - \Gamma_t^L$.

3.2. **The Origin of Disagreement.** We assume that the agent and the principal agree to disagree on the conditional distribution of S. In particular, we assume that, given that the principal has observed $T = t$, the agent's perceived probability of observing $S = s$ is altered by some amount b_t.

Assumption 3. *Regardless of the effort exerted, the agent has biased beliefs such that:*

$$\tilde{P}_{aa} = P_{aa} + b_a$$
$$\tilde{P}_{au} = P_{au} - b_a$$
$$\tilde{P}_{ua} = P_{ua} + b_u$$
$$\tilde{P}_{uu} = P_{uu} - b_u$$

[5]The model allows for two different interpretations. The acceptable or unacceptable performance may be either the agent's or the project's overall.

where

$$b_a \in [-P_{aa}, P_{au}] \quad b_u \in [-P_{ua}, P_{uu}].$$[6]

This modelling of the bias is more general than it seems at first glance and it allows us to study several different forms of bias observed in the lab. To distinguish among these cases we present the following definitions. Of course the list of cases we are going to study is not exhaustive. We start by stating the definition of agent optimism (and pessimism) in this model.

Definition 1. *The agent is* "optimistic" *if he overestimates the probability that his signal is acceptable given the realisation of the principal's signal, i.e., if b_a and b_u are positive.*

Definition 2. *The agent is* "pessimistic" *if he underestimates the probability that his signal is acceptable given the realisation of the principal's signal, i.e., if b_a and b_u are negative.*

The cases of agents with biases such that $b_u < 0 < b_a$ and $b_a < 0 < b_u$, that we define as "*trusty*" and "*skeptical*" respectively, are studied later on in section 6.

As described in its definition, the nature of optimism is for an agent to overestimate the probability that his performance is deemed "acceptable." In fact, denoting the biased probabilistic beliefs of the agent with $\tilde{\Pr}\{\cdot\}$, from basic probability theory we get:

$$\begin{aligned}\tilde{\Pr}\{S = a|\lambda^j\} =& \tilde{\Pr}\{S = a|T = a\} \Pr\{T = a|\lambda^j\} + \tilde{\Pr}\{S = a|T = u\} \Pr\{T = u|\lambda^j\} \\ =& \tilde{P}_{aa}\Gamma_a^j + \tilde{P}_{ua}\Gamma_u^j \\ =& \Pr\{S = a|\lambda^j\} + b_a\Gamma_a^j + b_u\Gamma_u^j\end{aligned}$$

which is increasing in both b_a and b_u for any $j = H, L$.[7]

A second aspect that needs attention is how the bias of the agent affects his beliefs about the correlation between the two signals. Overconfidence in calibration is another well documented psychological phenomenon. Most people, even experts, overestimate the precision of their estimates and forecasts (Oskamp, 1965; Fischhoff, Slovic, and Lichtenstein, 1977; Lichtenstein, Fischhoff, and Phillips, 1982; Wallsten, Budescu, and Zwick, 1993; Barberis and Thaler, 2003). Overconfidence is important in personal and business decisions (e.g. Russo and Schoemaker, 1992; Grubb, 2009). Overconfidence also matters for investment and financial decisions. Daniel, Hirshleifer, and

[6]The posed boundaries are needed for all $\tilde{P}_{ts} \in [0, 1]$ to hold.

[7]An alternative formulation for the bias would be to simply let

$$\tilde{\Pr}\{S = a|\lambda^j\} = \Pr\{S = a|\lambda^j\} + b.$$

An alternative definition of optimism emerges. One could, in fact, define an optimistic agent as one where $b_a\Gamma_a^j + b_u\Gamma_u^j = b > 0$. Our formulation and definition, however, is more general and allows for the study of more complicated beliefs.

Subrahmanyam (1998, 2001)Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) show that overconfidence can lead to excess volatility and to predictability of stock returns. Scheinkman and Xiong (2003) show that it can lead to financial bubbles. While the majority of individuals are overconfident, a minority is underconfident.

Definition 3. *The agent is* overconfident *if he overestimates the correlation between signals, i.e., if*

$$\tilde{\gamma}^j_{aa}\tilde{\gamma}^j_{uu} - \tilde{\gamma}^j_{au}\tilde{\gamma}^j_{ua} \geq \gamma^j_{aa}\gamma^j_{uu} - \gamma^j_{au}\gamma^H_{ua} \quad \Rightarrow b_a > b_u. \tag{1}$$

Definition 4. *The agent is* underconfident *if he underestimates the correlation between signals, i.e., if* $b_u > b_a$.

An agent's level and direction of confidence has implication on the *believed direction* of the correlation between signals. Given Assumption 2, an overconfident agent always believes signals to be positively correlated. An underconfident agent, instead, may underestimate the correlation between signals to the point of believing that T and S are negatively correlated. Given Lemma 1, an effort level λ^j, biases b_a and b_u imply the following for every t:

$$\tilde{\gamma}^j_{ta} = \tilde{P}_{ta}\Gamma^j_t = (P_{ta} + b_t)\Gamma^j_t$$

$$\tilde{\gamma}^j_{tu} = \tilde{P}_{tu}\Gamma^j_t = (P_{tu} - b_t)\Gamma^j_t$$

Lemma 2. *Given Assumption 2, when an agent has a bias that satisfies:*

$$b_u - b_a \leq P_{aa} - P_{ua}, \tag{2}$$

he believes signals to be positively correlated, i.e., $\tilde{\gamma}^j_{aa}\tilde{\gamma}^j_{uu} - \tilde{\gamma}^j_{au}\tilde{\gamma}^j_{ua} \geq 0$.

If (2) fails, a biased agent has beliefs that satisfy $\tilde{\gamma}^j_{aa}\tilde{\gamma}^j_{uu} - \tilde{\gamma}^j_{au}\tilde{\gamma}^j_{ua} < 0$ and expects signals to be negatively correlated.

To complete our classification of the types of bias, notice that an optimistic (pessimistic) agent can be either over or underconfident depending on the parameters.[8]

3.3. **The Principal's Effort Implementation Problem.** In the model, a contract is a set $\{w_{ts}, c_{ts}\}_{t,s\in\{a,u\}}$, where both the agent's compensation, c, and the principal's dollar cost of employing the agent, w, may depend on the realization of T and S. The principal is assumed to be perfectly informed about the agent's biased beliefs. The objective of the principal is to incentivize the agent to exert the level of effort that maximizes profits. Following Grossman and Hart (1983), we know that this problem can be divided in two steps. First, deriving the minimum cost $E^*(w|\lambda)$ of implementing

[8]We show later on that, given our assumptions, the only type of agent who is always overconfident is the trusty type. Further, optimistic, pessimistic, and skeptical agents can all believe that correlation is negative, under some parameter conditions.

a certain λ and then solving $\max_\lambda \lambda Y - E^*(w|\lambda)$. The rest of this paper is focused on minimizing the cost of implementing the high level of effort for different types of agents.

By the revelation principle, it is sufficient to consider only contracts where both parties have an incentive to reveal their private information in equilibrium. Hence, the principal faces the following constrained minimization problem:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \tag{3}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \bar{u} \tag{PC}$$

$$\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}^L_{ts} - V(\lambda^L) \tag{IC}$$

$$w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} \leq w_{ua}\gamma^H_{aa} + w_{uu}\gamma^H_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \leq w_{aa}\gamma^H_{ua} + w_{au}\gamma^H_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^H_{aa} + c_{ua}\tilde{\gamma}^H_{ua} \geq c_{au}\tilde{\gamma}^H_{aa} + c_{uu}\tilde{\gamma}^H_{ua} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^H_{au} + c_{uu}\tilde{\gamma}^H_{uu} \geq c_{aa}\tilde{\gamma}^H_{au} + c_{ua}\tilde{\gamma}^H_{uu} \tag{TR^u_A}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

The first two constraints are the classical participation and incentive compatibility constraint. They ensure that the agent is willing to accept the contract, (PC), and to exert high effort instead of low effort, (IC). Constraints (TR^t_P) are called *truthful reporting constraints for the principal*, they ensure that she is willing to truthfully report t when she observes $T = t$. Similarly, (TR^s_A) are the truthful reporting constraints for the agent, they ensure that he is willing to truthfully report s when he observes $S = s$. Last, is a set of four constraints that ensure limited liability on the side of the agent $(c_{ts} \geq 0)$ and feasibility $(w_{ts} \geq c_{ts})$.

Before deriving the optimal contract for the different types of agent, let us state the final assumption of our model. We require $\bar{u}$ to be small enough. This assumption implies that (PC) is satisfied and improves the tractability of the problem.

Assumption 4. *Let*

$$\bar{u} \leq \frac{V(\lambda^H)\Gamma^L_a - V(\lambda^L)\Gamma^H_a}{\Delta\Gamma_a} \tag{4}$$

Given Assumption 4, and the limited liability assumption, the (IC) implies that the (PC) is satisfied. Therefore we disregard the (PC) in the solution of the problem and check that it holds afterwards. We present this in Corollary 1.

$$
\begin{aligned}
&\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H && (5)\\
&\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^L - V(\lambda^L) && (IC)\\
&\qquad w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H \leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H && (TR_P^a)\\
&\qquad w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H && (TR_P^u)\\
&\qquad c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H && (TR_A^a)\\
&\qquad c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H && (TR_A^u)\\
&\qquad w_{ts} \geq c_{ts} \geq 0 \qquad \forall t,s \in \{a,u\}. && (LL_{ts})
\end{aligned}
$$

Before splitting the analysis for the resulting optimal contract for each type of biased agent, we present a set of findings on problem (5) which are valid for biased as well as for unbiased agents.

3.4. **Basic Features of Optimal Contracts.** In order for the truthful reporting constraints to hold, it cannot be always optimal for a party to report a certain signal realization. To see this, consider an agent who observes $S = a$. If both $c_{aa} \geq c_{au}$ and $c_{ua} \geq c_{uu}$ with at least one holding with inequality, it is always optimal for the agent to report a, regardless of the actual realization. Hence, in order for the (TR_A) constraints to hold, the two inequalities cannot have the same (strict) sign. A similar discussion holds for the principal and wages. Since the principal wants to pay the lowest possible wage, the direction of the inequalities must to be such that the wages are the lowest when the signal realizations are identical, i.e., $t = s$, the most probable outcome given that signals are positively correlated. This produces the following Lemma.

Lemma 3. *Given Assumption 2, any optimal contract implementing high effort features either (i) $w_{ua} = w_{aa}$ and $w_{au} = w_{uu}$ or (ii) $w_{ua} > w_{aa}$ and $w_{au} > w_{uu}$.*

Similarly, since the agent wants to obtain the highest possible compensation, if he believes signals are positively correlated, then the direction of the inequalities must to be such that the compensations are the highest when $t = s$, the most probable believed outcome.

Lemma 4. *If the agent believes signals are positively correlated, i.e. (2) holds, then any optimal contract implementing high effort features either (i) $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$ or (ii) $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$.*

These first two Lemmas are already enough for us to state the first Proposition of the model, which confirms one of the main results of MacLeod (2003) for an agent who believes signals are positively correlated, namely, that unless there is a deadweight loss it is impossible to implement high effort.

Proposition 1. *If the principal wishes to implement high effort and the agent believes signals are positively correlated, then there ought to exist at least one combination of realizations of* t *and* s *where* $w_{ts} > c_{ts}$.

To understand fully Proposition 1 notice that intuitively the principal always has the incentive to report that the performance of the project (and of the agent) is unacceptable, while the agent always has the incentive to report the opposite. If the principal and the agent play a constant sum game, these incentives are the only ones present and truthful reporting becomes impossible. We define the expected deadweight loss from using a subjective performance evaluation contract that implements high effort as $\sum_{ts}(w_{ts} - c_{ts})\gamma^H_{ts}$.

As one of the main results of our paper shows, however, Proposition 1 does not always hold for an agent who believes signals are negatively correlated. The basic characteristics of the contract, in fact, change as outlined in Lemma 5 below and produce the findings studied in section 4.2 and following.

Lemma 5. *If the agent believes signals are negatively correlated, i.e., (2) fails to hold, then any optimal contract implementing high effort features either (i)* $c_{aa} = c_{au}$ *and* $c_{uu} = c_{ua}$ *or (ii)* $c_{aa} < c_{au}$ *and* $c_{uu} < c_{ua}$.

Lemma 4 shows that if the agent believes signals are positively correlated, then the principal might opt for designing an optimal contract where, taking as given her signal realization, the compensation is higher in the agreement cases than in the disagreement cases, i.e., a contract with $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$. Lemma 5 shows that the opposite happens when the agent believes signals are negatively correlated. In this case the principal might opt for designing a contract where, taking as given her signal realization, the compensation is higher in the disagreement cases than in the agreement cases, i.e., a contract with $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$. This follows the exact opposite intuition of Lemma 4.

4. Optimal Contracting with an Optimistic Agent

In this section we derive the optimal contract for an optimistic agent. We separate the analysis into two subcases: an optimistic agent who believes signals are positively correlated and one who believes signals are negatively correlated.

4.1. **Optimistic Agent who Believes Signals are Positively Correlated.** We start by considering the case of an optimistic agent who believes signals are positively correlated. This happens when

$$b_a > 0,\ b_u > 0,\ \text{and}\ b_u - b_a \leq P_{aa} - P_{ua}.$$

Note that an optimistic agent who believes signals are positively correlated is underconfident when $b_a < b_u$ and overconfident when $b_a > b_u$.

In order to solve problem (5) when the agent believes signals are positively correlated we present a set of Lemmas in the appendix that select the binding constraints for this case and reduce the choice variables of the problem to simply: c_{aa} and c_{au}. All together this reduces the problem to:

$$\begin{aligned}
\min_{c_{aa},c_{au}} \; & c_{aa}[(\gamma_{aa}^H)^2\tilde{\gamma}_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H + \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H] \\
& + c_{au}\left(\gamma_{aa}^H\gamma_{au}^H\tilde{\gamma}_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H\right) && (6) \\
\text{s.t.} \quad & c_{aa}\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + c_{au}\left(\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) \geq \Delta V && (IC) \\
& c_{aa} \leq \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)c_{au} && (7) \\
& c_{aa} \geq c_{au}, && (8)
\end{aligned}$$

where $\Delta V = V(\lambda^H) - V(\lambda^L)$. The last two conditions ensure that (TR_P^u) and (TR_A^a) hold respectively. If they hold with equality, the corresponding constraint is binding.

The next Proposition presents a condition that selects the binding constraints of (6). Compared to the Lemmas in the appendix, this Proposition is far more important. Combined with Proposition 3, it presents a result original to our model. That is, as we show later, the existence of a new contract where the principal's wage cost is only determined by the agent's performance evaluation, as opposed to the baseline contract in the literature where the wage is determined by both parties' performance evaluations.

Proposition 2. *Let the agent believe that signals are positively correlated. If the agent has beliefs that satisfy:*

$$b_u \leq P_{uu} - \frac{(P_{au} - b_a)\left(1 - \Gamma_a^H\right)\Gamma_a^H\left(P_{aa} - P_{ua}\right)}{\left(b_a - P_{au}\Gamma_a^H\right)\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)} \tag{9}$$

then the optimal contract implementing high effort features $c_{aa} > c_{au}$, (TR_A^a) slack and (TR_P^u) binding. If the agent has beliefs that violate (9), then the optimal contract implementing high effort features $c_{aa} = c_{au}$, (TR_A^a) binding and (TR_P^u) slack.

Proposition 2 follows from a graphical analysis of the problem. Figure 1 below shows the three constraints binding in (c_{au}, c_{aa}) space and highlights the set of contracts satisfying all constraints of (6) — and therefore of (3).

In order to understand whether at optimum it is the (TR_A^a) or the (TR_P^u) that bind, and therefore where does the optimal contract lie in Figure 1, we study the sign and magnitude of the slope of the iso-costs and the (IC). Hence, Proposition 2 shows that the optimal contract lies either at point X or Y of Figure 1 depending on the how do the slope of the (IC) and of iso-cost compare.

From this analysis we can also derive the optimal contract offered to an unbiased agent, which we refer to as the *Baseline Performance Evaluation* (BPE) contract $\{w_{ts}^*, c_{ts}^*\}_{t,s=a,u}$.

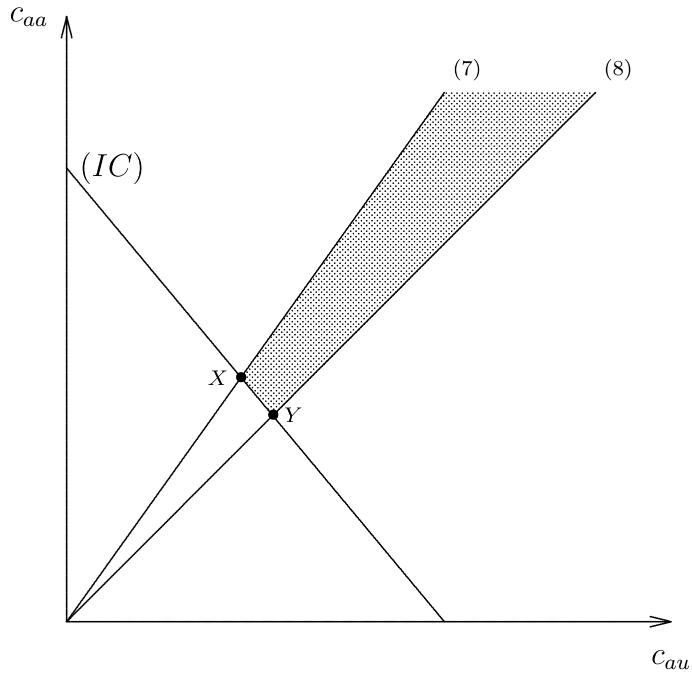

FIGURE 1. The shaded area represents the set of contracts satisfying all the constraints of (3).

Proposition 3 (BPE Contract)**.** *If the agent has unbiased beliefs, then the optimal contract implementing high effort is given by:*

$$\begin{array}{llll} w^*_{aa} = c^*_{aa} & w^*_{au} = c^*_{aa} & w^*_{uu} = 0 & w^*_{ua} = \frac{c^*_{aa}}{P_{aa}} \\ c^*_{aa} = \frac{\Delta V}{\Delta \Gamma_a} & c^*_{au} = c^*_{aa} & c^*_{uu} = 0 & c^*_{ua} = 0. \end{array}$$

The BPE contract features:

(i) *a wage that depends on both parties' performance evaluations;*
(ii) *a compensation that only depends of the principal's performance evaluation (a compensation that is independent of the agent's performance evaluation);*
(iii) *a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent;*
(iv) *no wage and no compensation when both parties deem the performance unacceptable;*
(v) *no compensation when the principal deems the performance unacceptable.*

The above replicates the standard result of the literature with unbiased agents (à la MacLeod, 2003) and it provides us with a basis of comparison for the contracts derived hereafter. The key features of the BPE contract are as follows. First, the principal's wage cost depends on both parties' performance evaluations. Second, the agent's compensation depends only on the principal's performance evaluation. Third, there is a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent.

We now present a set of results that show how, in the presence of an optimistic agent, the principal may find it optimal to offer the agent either the baseline contract or a new contract that makes different use of information and takes advantage of the agent's bias — which we call the *Agent's Performance Evaluation* (APE) contract. The APE contract is original to the present model.

Proposition 4. *If the agent is optimistic, believes signals are positively correlated, and has beliefs that violate* (9)*, then the optimal contract implementing high effort is given by* $\{w'_{ts}, c'_{ts}\}_{t,s=a,u}$ *where both wages and compensations equal the ones of the BPE contract, i.e.,* $c'_{ts} = c^*_{ts}$ *and* $w'_{ts} = w^*_{ts}$ $\forall t, s = a, u$.

Proposition 4 shows that the optimal contract for an optimistic agent is the same as that for an unbiased agent when the optimistic agent believes signals are positively correlated and his bias is small.

Now suppose (9) holds. In this case the APE $\{w^\dagger_{ts}, c^\dagger_{ts}\}_{t,s=a,u}$ is set at optimum. This new contract differs qualitatively from the BPE contract as we discuss below.[9]

Proposition 5 (APE Contract)**.** *If the agent is optimistic, believes signals are positively correlated, and has beliefs that satisfy* (9)*, then the optimal contract implementing high effort* $\{w^\dagger_{ts}, c^\dagger_{ts}\}_{t,s=a,u}$ *is given by:*

$$\begin{array}{llll} w^\dagger_{aa} = c^\dagger_{aa} & w^\dagger_{au} = c^\dagger_{au} & w^\dagger_{uu} = c^\dagger_{au} & w^\dagger_{ua} = c^\dagger_{aa} \\ c^\dagger_{aa} = c^\dagger_{au}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right) & c^\dagger_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & c^\dagger_{uu} = c^\dagger_{au} & c^\dagger_{ua} = 0. \end{array}$$

The APE contract features:

(i) *a wage that only depends of the agent's performance evaluation (a wage that is independent of the principal's performance evaluation);*

(ii) *a compensation that depends on both parties' performance evaluations;*

(iii) *a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent;*

(iv) *a wage and a compensation when both parties deem the performance unacceptable;*

(v) *no compensation when the principal deems unacceptable a performance deemed acceptable by the agent.*

Proposition 5 shows that if the optimistic agent believes signals are positively correlated and his bias is large, then the optimal contract is very different from the BPE

[9] Notice that, if condition (9) holds with equality, the slopes of the IC and isocosts are identical and the problem has many solutions. In particular any point lying between X and Y in Figure 1 solves problem (6). At this point of indifference, we assume the principal sets up a APE contract.

contract described in Proposition 4. The key differences are as follows. First, the principal's wage cost depends only on the agent's performance evaluation. Second, the agent's compensation depends on both parties' performance evaluations. Third, the agent's optimism reduces the deadweight loss associated with SPE. Next we provide the economic intuition behind these key differences.

In the BPE contract in Proposition 4 the principal's wage cost depends on both parties' performance evaluations. Moreover, the only role played by the agent's performance evaluation signal is to provide incentives for truthful revelation by the principal through the imposition of a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent. In other words, the threat of conflict ensures that the principal has an incentive to reveal favorable observations that result in higher compensation to the agent. Hence, the possibility of a deadweight loss makes the agent's performance evaluation signal valuable in the sense of Hölmstrom (1979).[10] In contrast, in the APE contract in Proposition 5 the principal's wage cost only depends on the agent's performance evaluation signal. The intuition behind this result is as follows.

In the APE contract the agent's performance evaluation signal is still valuable in the sense of Hölmstrom (1979). However, when the optimistic agent believes signals are positively correlated and his bias is large, i.e., the optimistic agent's beliefs satisfy (9), the principal can decrease her expected cost of implementing high effort by increasing the correlation between the wage and the agent's signal. The principal can do that by increasing the wage when both parties deem the performance acceptable, a state overestimated by the agent, and lowering the wage when the principal deems acceptable a performance deemed unacceptable by the agent, a state underestimated by the agent. The principal exploits the agent's bias maximally by making the wage depend only on the agent's signal.

In the BPE contract in Proposition 4 the agent's compensation only depends on the principal's performance evaluation signal. This result follows from the fact that the principal's signal is more informative than that of the agent, combined with the linearity of the incentive constraints.[11] In contrast, in the APE contract in Proposition 5 the agent's compensation depends on both parties' performance evaluation signals. On the one hand, we still have the old effect, namely, the fact that principal's signal is more informative than that of the agent and therefore the principal wants to use it to compensate the agent. However, for an optimistic agent with beliefs that satisfy (9), the agent's signal, while being less informative than that of the principal, allows the principal to exploit the agent's bias by increasing the agent's compensation when

[10] According to Hölmstrom (1979), pp.83: "A signal y is said to be *valuable* if both the principal and the agent can be made strictly better off with a contract of the form $s(x, y)$ than they are with a contract of the form $s(x)$."

[11] Here the term "more informative" is used in the sense defined by Blackwell et al. (1951, 1953).

both parties deem the performance acceptable, a state overestimated by the agent, and lowering the agent's compensation when the principal deems acceptable a performance deemed unacceptable by the agent, a state underestimated by the agent. The optimal compensation in Proposition 5 results from this trade-off between informational efficiency and exploitation of the agent's bias. We discuss more about the information efficiency of each contract in section 7.

In order to fully describe and study the optimal contract we now present a graphical representation of the feasible portion of (b_a, b_u) space for an optimistic agent. Figure 2 below identifies the type of contract an optimistic agent is offered for any (feasible) value of his bias. The $b_a > b_u$ condition (that ensures overconfidence) is represented by the dotted 45° line. From the Figure, we see that the area where the APE is set optimally crosses the $b_a = b_u$ line. In the proof of Proposition (5) we formally prove the shape of the area where the APE contract is set up.

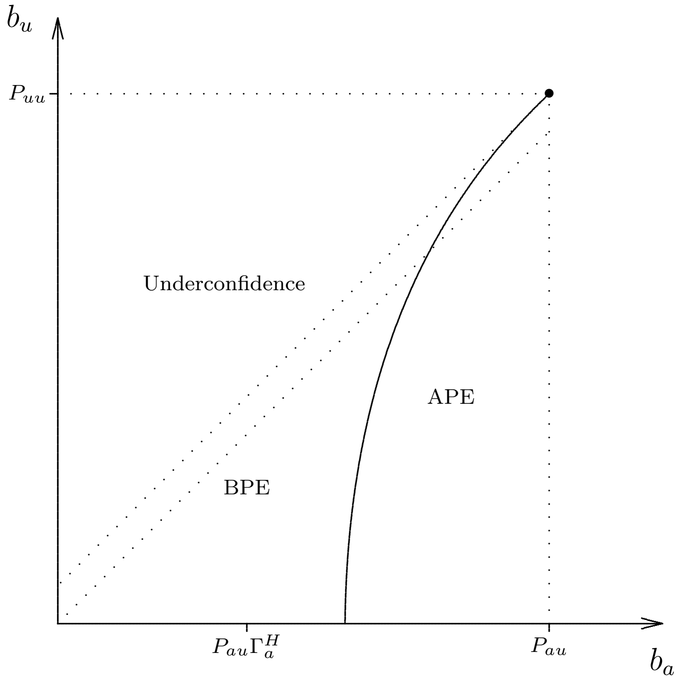

FIGURE 2. The area delimited by the solid curve on the right in the figure identifies the portion of the parameter space where a contract of the type described in Proposition 5 is set optimally. That is, when the presence of optimism generates a new contract compared to the case of an unbiased agent. The two dotted lines crossing the quadrant represent the condition for overconfidence (the one below) and the condition for beliefs to satisfy (2) (the one above). This specific graph was obtained for $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. Its shape, however generalises to all feasible parameter values. The size and position of the area where the APE contract is offered is unaltered.

First of all, notice that when $b_u = 0$ and $b_a = P_{au}$ the agent believes that there is no chance that he will receive a signal of an unacceptable performance conditional on the principal deeming it acceptable, i.e., he believes that conditional on $T = a$, $S = u$ is not possible. Upon observing $S = u$, he believes *fully* his signal's realization and thinks that the principal has observed exactly the same. Intuitively, this is the case where the principal can exploit the most the agent's optimism. The agent, in fact, has no "suspicion" that the principal may have observed a, while in reality this may very well be the case. Hence, as we show in section 7, the agent's compensation for an acceptable performance becomes cheaper with an APE contract compared to a BPE contract ($c^{\dagger}_{au} < c^{*}_{au}$).

4.2. **Optimistic Agent who Believes Signals are Negatively Correlated.** Consider now the case of an optimistic agent who believes signals are negatively correlated. This happens when

$$b_a > 0,\ b_u > 0,\ \text{and}\ b_u - b_a > P_{aa} - P_{ua}.$$

Note that an optimistic agent who believes signals are negatively correlated is always underconfident since $b_u - b_a > P_{aa} - P_{ua}$ and $P_{aa} > P_{ua}$ imply $b_u > b_a$.

When an optimistic agent believes the signals are negatively correlated, the compensation scheme of the contract has to subdue to different properties in order to satisfy truthful reporting. In the previous section we proved, in fact, that Lemma 4 does not hold any longer. On the contrary, the optimal contract must satisfy Lemma 5.

In order to solve problem (5) when the agent believes signals are negatively correlated we present a set of Lemmas in the appendix that select the binding constraints for this case and reduce the problem to:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad c_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} \tag{10}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma^H_a\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_u \tag{IC}$$

$$w_{au}\frac{P_{au}}{P_{aa}} \leq (w_{ua} - c_{aa}) \tag{TR^a_P}$$

$$(w_{ua} - c_{aa}) \leq w_{au}\frac{P_{uu}}{P_{ua}} \tag{TR^u_P}$$

$$c_{au} \geq c_{aa} \tag{TR^u_A}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

$$w_{au} \geq c_{au} \geq 0. \tag{LL_{au}}$$

The study of the solution of (10) is longer and more complicated than the solutions to (6). First of all, notice that this time we have $c_{uu} = 0$ (Lemma 14 in appendix A). When the agent believes signals to be negatively correlated the "disagreement" payoff is given by the cases where the realization of T and S are the same. In particular if the agent observes $S = u$, he believes that the principal has observed $T = a$ and is not easily convinced that $T = u$ instead.

The first implication of the above, is that the proof of Proposition 1 does not hold any longer. That is, under some conditions, the bias of an agent who believes signals to be negatively correlated is such that the existence of a deadweight loss is not a necessary condition for the implementation of high effort any longer. Further, as we show later, there exists a portion of the parameter space where the optimal contract does not, in fact, feature a deadweight loss. This is, however, not the case for an optimistic agent. While there may exist equilibrium contracts different from the standard one for an agent with beliefs violating (2), they are never optimal when the agent is optimistic.

Proposition 6. *If the agent is optimistic and believes signals are negatively correlated, then he is assigned the BPE contract.*

In the proof of Proposition 6 we show how the conditions for potential new contracts to be assigned can never be satisfied if the agent is optimistic and believes signals are negatively correlated.

5. Optimal Contracting with a Pessimistic Agent

In this section we derive the optimal contract for a pessimistic agent. We separate the analysis into two subcases: a pessimistic agent who believes signals are positively correlated and one who believes signals are negatively correlated.

5.1. **Pessimistic Agent who Believes Signals are Positively Correlated.** A pessimistic agent who believes signals are positively correlated has beliefs such that

$$b_a < 0,\, b_u < 0, \text{ and } b_u - b_a \leq P_{aa} - P_{ua}.$$

Note that Lemmas 3 and 4 and Proposition 1 hold for this case. As the next Proposition states, however, no pessimistic agent who believes signals are positively correlated is assigned the APE contract.

Proposition 7. *A pessimistic agent who believes signals are positively correlated is always offered the BPE contract.*

5.2. **Pessimistic Agent who Believes Signals are Negatively Correlated.** A pessimistic agent who believes signals are negatively correlated has beliefs such that

$$b_a < 0,\, b_u < 0, \text{ and } b_u - b_a > P_{aa} - P_{ua}.$$

Following up on the discussion started in section 4.2 we derive the optimal contract for a pessimistic agent who believes signals are negatively correlated. We show how it can take two new forms, the *Performance Evaluation Disagreement Deadweight Loss*

(PED-DL) contract and the *Performance Evaluation Disagreement No Deadweight Loss* (PED-NDL) contract. We are going to present each new contract in a Proposition.[12]

As already anticipated in section 4.2, when the agent believes signals are negatively correlated, the existence of a deadweight loss is not a necessary condition for the implementation of high effort any longer. It can, however, still be take place under certain conditions, as the next result shows.

Proposition 8 (PED-DL Contract)**.** *If the agent is pessimistic, believes signals are negatively correlated, and has beliefs that satisfy*

$$b_u P_{au} \Gamma_u^H - b_a P_{aa} \Gamma_a^H \geq P_{aa}^2 \Gamma_a^H - P_{au} P_{ua} \Gamma_u^H \tag{11}$$

and

$$b_a < -P_{aa} \Gamma_A^H, \tag{12}$$

then the optimal contract implementing high effort $\{\hat{w}_{ts}, \hat{c}_{ts}\}_{t,s=a,u}$ *is given by:*

$$\begin{array}{llll} \hat{w}_{aa} = 0 & \hat{w}_{au} = \hat{c}_{au} & \hat{w}_{uu} = 0 & \hat{w}_{ua} = \frac{P_{au}}{P_{aa}} \hat{c}_{au} \\ \hat{c}_{aa} = 0 & \hat{c}_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & \hat{c}_{uu} = 0 & \hat{c}_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \hat{c}_{au}. \end{array}$$

The PED-DL contract features:

(i) *a wage that depends on both parties' performance evaluations;*
(ii) *a compensation that depends on both parties' performance evaluations;*
(iii) *a deadweight loss when the principal deems unacceptable a performance deemed acceptable by the agent (unless* (11) *holds with equality);*
(iv) *a wage and a compensation when the parties disagree on their performance evaluations (no wage and no compensation when the parties agree on their performance evaluations).*

This result show that if the pessimistic agent believes signals are negatively correlated and has a large bias, then the optimal contract is very different from the BPE contract described in Proposition 4. The key differences are as follows. First, the principal's wage cost and the agent's compensation depend on both parties' performance evaluations. Second, there is a wage and a compensation only when the parties *disagree* on their performance evaluations. Next we provide the economic intuition behind the PED-DL contract.

When the agent believes that signals are negatively correlated two very similar and connected effects take place: (i) he believes "au" and "ua" more probable than "aa" and "uu" (at least jointly) and (ii) his believed most probable events are the symmetric opposite of the ones believed by the principal. Hence, it is straightforward to see why a pessimistic agent who believes that signals are negatively correlated never accepts a

[12]The proofs are presented as one in the appendix.

APE contract. First, he rarely expects to obtain $c_{aa}^{\dagger}$. Second, he is not willing any longer to accept a contract that features $c_{ua} = 0$. Therefore, in order for the PED-DL contract to be optimal, the latter has to be feasible and has to implement high effort with a lower cost than then BPE contract.

Similarly to the APE contract, the principal can take advantage of the agent's bias in order to decrease the expected wage paid. While the agent expects the cases of $(T, S) = (u, a)$ $(T, S) = (a, u)$ to be the most likely, the principal knows that that is not the case. She is therefore happy to offer the agent a positive c_{ua} and a larger c_{au} (compared to the BPE case) in exchange for a lower c_{aa} and/or c_{uu}.[13]

Hence, just as in the case of the APE contract, the principal is "speculating" on the disagreement by increasing the compensation of the agent in states he wrongly deems more probable and decreasing it in states the agents wrongly deems less probable. The fact that they do not disagree only on the extent of the correlation any longer but now also on the direction of it, opens up to a "stronger" manipulation of the standard contract, compared to the switch from BPE to APE. This brings the result of $c_{aa} = c_{uu} = 0$.

Obviously, the above has to be feasible, i.e. the agent has to be biased enough to accept such a manipulation compared to the BPE contract, and optimal, i.e. the PDE contract has to implement high effort at a lower cost. These are precisely the meanings of condition (11) and (12). Condition (11) requires the agent to be biased enough to accept a PDE contract while (12) requires the agent to be biased enough for the principal to find it optimal to offer a PDE contract. To better understand the meaning of, and intuitions behind the, conditions let us represent them in in (b_a, b_u) space.

Start by noticing that (11) may imply (12) under some parameter conditions, but the reverse is never true. Figure 3 represents the two conditions for the case of $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.[14]

Condition (12) poses a restriction only on b_a, requiring it to be negative and low enough. Intuitively, the "more negative" is b_a the lower is $\tilde{P}_{aa}$ and the less the agent expects $(T, S) = (a, a)$ to take place. When (12) holds, the bias of the agent is such that the principal has the incentive to speculate as described above. In other words, $\tilde{P}_{au}$ is large enough for her to be able to offer a new contract with a lower c_{au} and higher c_{aa} satisfying all the constraints of the problem. Whether the agent sees through this "deception" or not, depends on the restrictions posed on b_a and b_u by (11).

[13]We prove that this is indeed the case in Proposition 10.

[14]Similarly to what we discussed in section 4.1, if condition (12) holds with equality, the slopes of the IC and isocosts are identical and the problem has many solutions. In particular any point lying between X and Y in Figure 12, presented in the proof, solves the principal's problem. At this point of indifference, we assume the principal sets up a PED-DL contract.

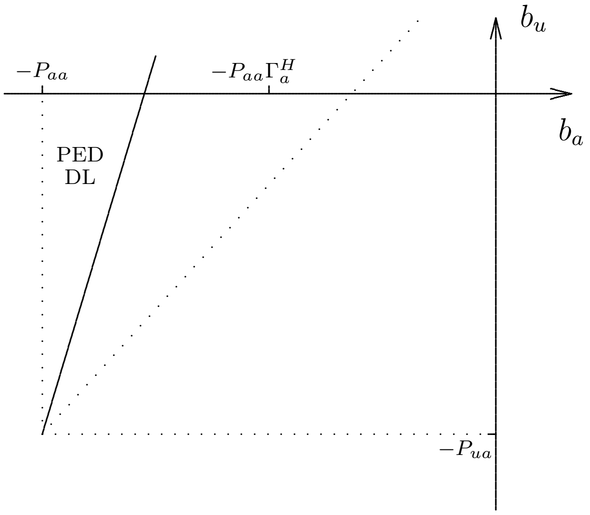

FIGURE 3. In the Figure we represent conditions (11) and (12). Together they define the area where a pessimistic underconfident agent is assigned an PED-DL contract. The dotted line crossing the quadrant represents (2). In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

While a negative enough b_a ensure the profitability of a PDE contract, the agent may reject such a contract if his b_u is negative or in general low enough. To see this, notice that the smaller is b_u, the less the agent expects $(T, S) = (u, a)$ to take place, an the more he expects $(T, S) = (u, u)$ to take place. Since the $(T, S) = (u, u)$ grants him a zero compensation under a PDE contract, the latter becomes unfeasible. Hence, for a given (negative) b_a the agent has to have a large enough b_u. To see why this is reflected in the Figure, consider the area where the PED-DL is set in Figure 3 (and also in Figure 5 and 6 in section 6 below). Notice that it is at its largest when b_u is large and positive and b_a is large and negative. This is precisely when the disagreement on correlation is at its maximum since $\tilde{P}_{aa}$ and $\tilde{P}_{uu}$ are close to zero.

Before going ahead, notice that a key aspect of this contract relies in the full implementation of both signal T and signal S. The disagreement about the correlation between the signals allows the principal to design contracts that take advantage of both information sources. This is confirmed in the next type of contract as well.

As we mentioned already, this paper shows how the classical result of Proposition 1 does not necessarily hold in the presence of an agent who (wrongly) believes signals to be negatively correlated. This is originated by the disagreement on the direction of the correlation.[15] To see this, suppose the agent observes $S = a$ and that he believes signals to be negatively correlated. Two opposed effects take place. Clearly the agent

[15]It would, in fact still hold if the true correlation were negative and the agent and principal agreed on it.

would be very happy to hear the principal reporting $T = a$, but what happens if the principal reports $T = u$? On the one hand, the agent is upset because the principal deems his performance unacceptable, and therefore would like to punish her in general. On the other hand, however, the agent expects $T = u$ because she believes signals to be negatively correlated! So he is less prone to punish the principal because he is more convinced that $T = u$ is indeed the truth. Has already explained above, the principal takes advantage of this by setting $c_{aa} = 0$. When the agent reports $S = a$, he knows that if the principal reports $T = a$, he will get no compensation at all. This makes the agent (i) willing to report $S = a$ only when it is indeed true, (ii) less prone to punish the principal compared to the positive correlation case. Under some particular levels of bias, this effect is so strong that the presence of a deadweight loss case in the contract is not a necessary condition for its implementation any longer. This is highlighted in the following proposition.

Proposition 9 (PED-NDL Contract)**.** *If the agent is pessimistic, believes signals are negatively correlated, has beliefs that violate* (11) *but satisfy*

$$b_u P_{uu} \Gamma_u^H - b_a P_{ua} \Gamma_a^H > P_{aa} P_{ua} \Gamma_a^H - P_{uu} P_{ua} \Gamma_u^H \tag{13}$$

and

$$b_a \leq -P_{aa} \left(\frac{P_{ua} + b_u \Gamma_a^H}{P_{ua}(1 + \Gamma_u^H) + b_u} \right), \tag{14}$$

then the optimal contract implementing high effort $\{\hat{w}'_{ts}, \hat{c}'_{ts}\}_{t,s=a,u}$ *is given by:*

$$\begin{array}{llll} \hat{w}'_{aa} = 0 & \hat{w}'_{au} = \hat{c}_{au} & \hat{w}'_{uu} = 0 & \hat{w}'_{ua} = \hat{c}_{ua} \\ \hat{c}'_{aa} = 0 & \hat{c}'_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & \hat{c}'_{uu} = 0 & \hat{c}_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \hat{c}_{au}. \end{array}$$

The PED-NDL contract features:

(i) *a wage that depends on both parties' performance evaluations;*
(ii) *a compensation that depends on both parties' performance evaluations;*
(iii) *no deadweight loss;*
(iv) *a wage and a compensation when the parties disagree on their performance evaluations (no wage and no compensation when the parties agree on their performance evaluations).*

Proposition 9 shows that there exist PED contracts which do not involve a deadweight loss. Identifying the set of parameter values under which a PED-NDL contract is feasible and optimal is no easy task because of the several conditions behind it and the fact that none of them implies any of others for all parameter values. In Figure 4 below, we plot the area where a PED-NDL contract is feasible and optimal for $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

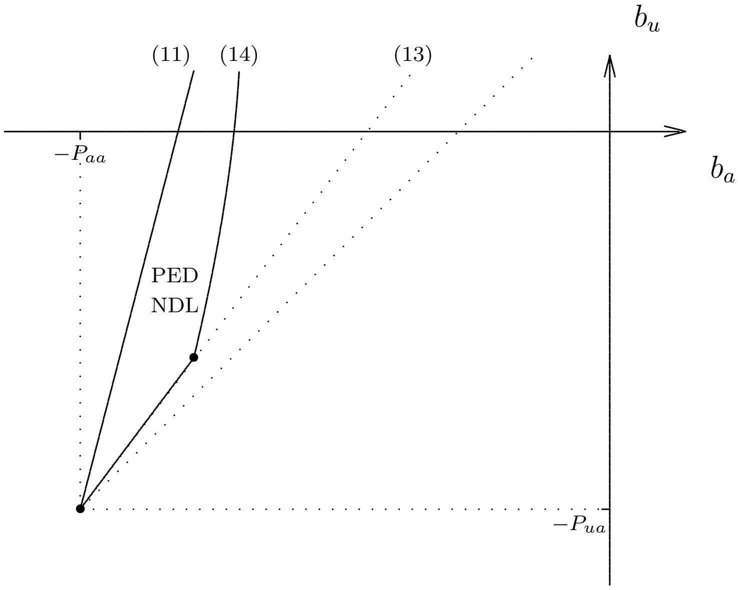

FIGURE 4. In the Figure we represent conditions (11), (13), and (14). Together they define the area where a pessimistic underconfident agent is assigned an PED-NDL contract. The bullet indicates the point where (14) becomes tighter than (13). The dotted line crossing the quadrant represents (2). In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

While condition (13) and (14) play the exact same role for a PDE-NDL contract as (11) and (12) do for a PDE-DL contract, the requirement of (11) to fail needs attention. In the proof of Proposition 9 we show how condition (11) determines whether the LL_{ua} or the TR_P^a is more stringent in problem (10). When (11) holds, the TR_P^a is more stringent and the contract must feature a deadweight loss. When it fails, the contract features no deadweight loss. In other words, condition (11) failing together with condition (13) identify an area where an agent who disagrees with the principal on the correlation of signals has a bias such that he does not need to be able to punish the principal under any realization of T and S.

It is possible to show that a PED-NDL contract may be unfeasible for all b_a and b_u under some parameter conditions.[16] This implies that the presence of a (particularly) biased agent may not be enough for the principal to be able to set up a contract without a deadweight loss. The proofs show that in any portion of the feasible (b_a, b_u) space for a pessimistic agent who believes signals are negatively correlated where none of the PED contracts is assigned, the BPE contract is assigned instead.

To conclude this section, let us state the promised Corollary to the Propositions of section 4 and 5, showing that the (PC) is satisfied by all the contracts derived when Assumption 4 holds.

[16]For example for the same parameters of Figure 4 but $\Gamma_a^H = 0.25$, PED-DL is the only feasible and optimal contract other than the BPE assigned to a pessimistic underconfident agent with beliefs that violate (2).

Corollary 1. *Given Assumption 4, all the potentially optimal contracts derived satisfy the (PC) constraint, which is therefore slack.*

6. Other Forms Of Biases — Trusty and Skeptical Agent

In this section we study two further possible cases of agent's type allowed by the model. We call *trusty*, for his nature to agree with the principal's view, an agent with $b_a > 0 > b_u$. On the other hand, we call *skeptical*, for his nature to disagree with the principal's view, an agent with $b_u > 0 > b_a$. We show in the following how a skeptical and a trusty agent may be offered only the type of contracts derived so far.

By studying the proofs of the Propositions proven so far, it is possible to see that none of the conditions derived change in the presence of a trusty or skeptical agents. This originates Figure 5 and Figure 6 where the entire parameter space (b_a, b_u) is partitioned in the areas where each of the contracts derived so far is optimal. While the condition for optimality of the APE contract has a clear shape (see the proof of Proposition 5) the existence and shape of the areas where the PED contracts are set optimally are not constant for every value of $\left(P_{aa}, P_{uu}, \Gamma_a^H\right)$. This is because of the nature of the conditions behind the optimal contract for an agent who believes signals are negatively correlated. We therefore present two possible alternative parameter configurations. In Figure 5 we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. In Figure 6 we assume assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.6, 0.5, 0.35)$.

Figure 5 shows a set of parameters that allows for the possibility of a contract featuring no deadweight loss to be assigned to a particularly biased agent. On the other hand, Figure 6 displays a situation where the correlation between signals (and the probability of the principal to deem the performance acceptable under high effort) are such that there is no type of agent that would accept a PED contract without a deadweight loss *and* exert high effort. A further difference between the two figures is that in Figure 5 condition (12) does not bind when delimiting the area where PED-DL is set, while it does in Figure 6.

7. Welfare and the Social Value of Biased Agents

In this section we present a welfare analysis and prove formally that some types of the agent's bias may be socially desirable. That is, compared to the BPE contract, they lead to new contracts that increase social welfare.

Since the BPE contract is the only contract of equilibrium for an unbiased agent, we compare the welfare and efficiency of the new contracts to the ones of the BPE contract. Further, since a APE is the only other potentially optimal contract for an

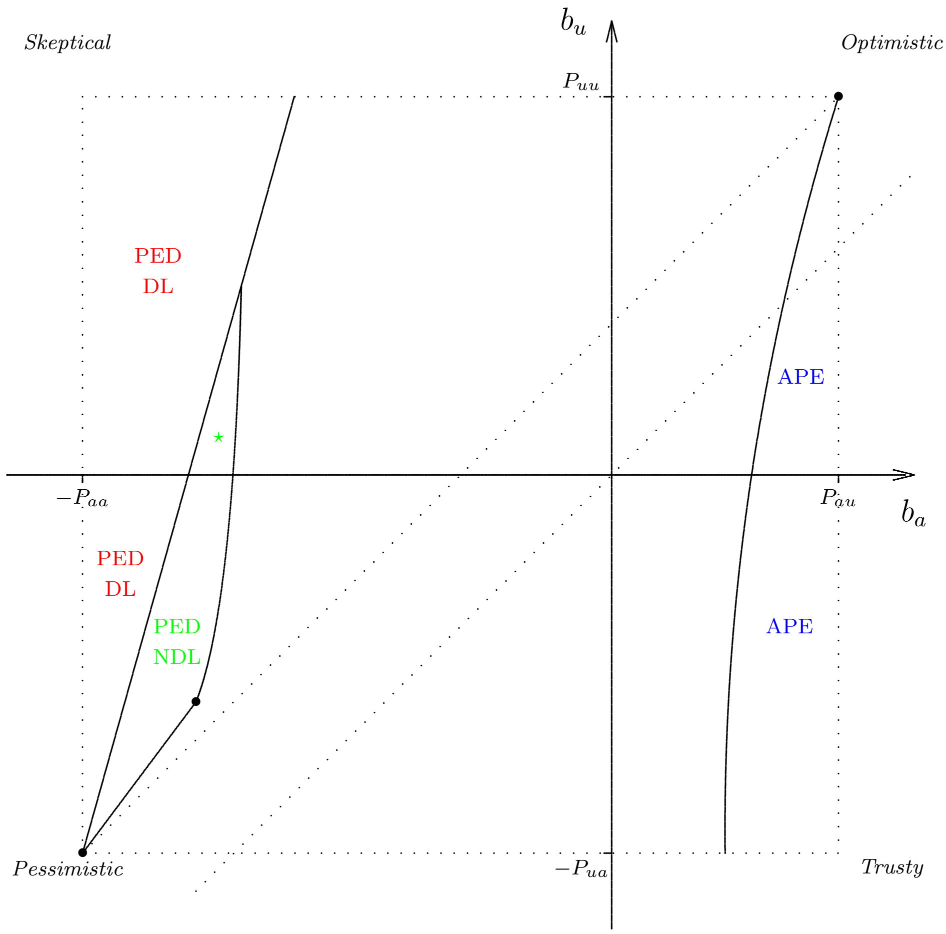

FIGURE 5. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. We highlight in colours the areas where each contract derived so far is set optimally. The BPE contract is set up in the areas without a coloured contract acronym. The bottom dotted line crossing the graph split the space in overconfident (below the 45°) and underconfident (above it). The top dotted line crossing the graph split the space between agents who believe signals to be positively correlated (below the line) and those who believe T and S are negatively correlated (above it) $\star$=PED-NDL.

optimistic or trusty agent and the PED contracts can only be set up for pessimistic or skeptical agents, the analysis is separated accordingly.

As stated in the appendix, the APE contract is possible only when the (IC) constraint is negatively sloped, the condition for which is given by $b_a \geq \Gamma_a - P_{aa}$. Hence, we have that $c_{au}^{\dagger} < c_{aa}^{*}$. The following Proposition compares the maximum compensations available for different types of agents.

Proposition 10. *Compensation $c_{aa}^{\dagger}$ is the maximum compensation available to either an optimistic agent or to a trusty agent. Compensation $\hat{c}_{au}$ is the maximum compensation available to either a pessimistic agent or to a skeptical agent if $\tilde{\gamma}_{aa}^{H} > \tilde{\gamma}_{ua}^{H}$. Otherwise it is $\hat{c}_{ua}$.*

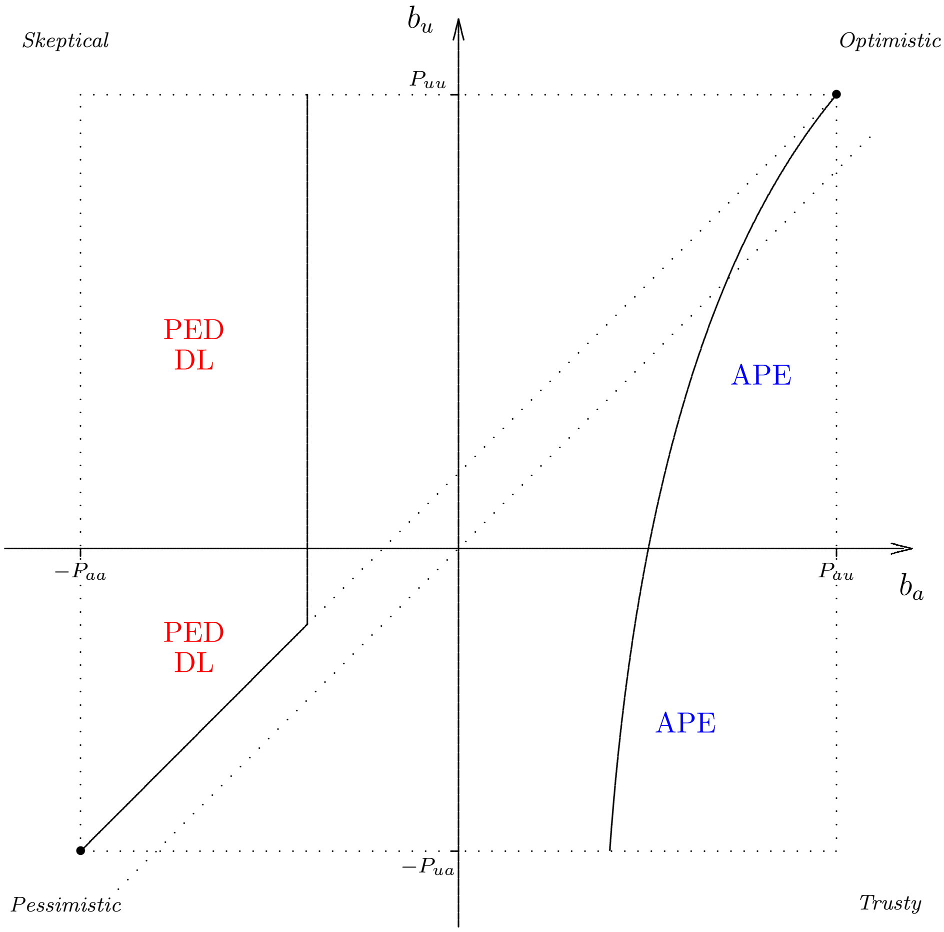

FIGURE 6. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.5, 0.6, 0.35)$. We highlight in colours the areas where each contract derived so far is set optimally. The BPE is set up in the areas without a coloured contract acronym. The bottom dotted line crossing the graph split the space in overconfident (below the 45°) and underconfident (above it). The top dotted line crossing the graph split the space between agents who believe signals to be positively correlated (below the line) and those who believe T and S are negatively correlated (above it).

When the agent believes signals are positively correlated and is optimistic enough for the principal to assign him a APE contract, the latter features a large compensation for the case of $T = S = a$, the probability of which, as discussed above, is overestimated by the agent. This creates an opportunity for the principal to take advantage of the agent's bias.[17] On the contrary, as we discussed in section 5.2 and 6, the agreements compensations (c_{aa} and c_{uu}) in the PED contracts are set to zero. Because of the wrong direction of the believed correlation between signals by the agent, the principal takes advantage of his bias by setting up a contract that features positive compensation and wages only in case of disagreement, which she knows are less probable than the rest.

[17]Hence the APE maybe connected to the idea of "exploitative" contracts in the literature on agents with biased beliefs (Eliaz and Spiegler, 2006, 2008; Foschi, 2017).

We continue the analysis by first comparing the BPE contract to the APE contract in the next Proposition.

Proposition 11. *Let $\tilde{E}(\cdot)$ denote the biased expectations of the agent. Given the BPE contract $\{w^*_{ts}, c^*_{ts}\}$ and the APE contract $\{w^\dagger_{ts}, c^\dagger_{ts}\}$, the following are true:*

(i) $E(w^*_{ts}) > E(w^\dagger_{ts})$ *whenever the APE is the contract of equilibrium.*

(ii) $E(c^*_{ts}) = \tilde{E}(c^*_{ts}) = \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a$.

(iii) $\tilde{E}(c^\dagger_{ts}) > \tilde{E}(c^*_{ts})$ *always.*

(iv) $E(c^\dagger_{ts}) > E(c^*_{ts})$ *whenever*

$$b_u \geq P_{uu}\frac{b_a(1+\Gamma^H_u) - P_{au}}{b_a - P_{au}\Gamma^H_a}. \tag{15}$$

Proposition 11 provides a set of intuitive conclusions. First of all, point (i) obviously states that for the principal to be willing to switch to an APE contract from a BPE one, it has to be optimal for her to do so. That is, she must be paying a lower expected wage. Point (ii) follows from the fact that in the BPE contract the agent's compensation does not depend on the agent's beliefs. Point (iii) shows that the agent would always be happy to be assigned the APE contract instead of the standard one. This is because the optimistic agent (and the trusty agent) overestimates the chances of obtaining $c^\dagger_{aa}$, that we show above is the largest possible compensation available among the ones in the BPE and APE contracts. Finally, point (iv) is by far the most interesting and important. It shows that, even though the principal would like to take advantage of the agent's biased beliefs, under some conditions, the APE contract is not exploitative after all. If conditions (15) holds, in fact, the contract not only allows the principal to pay a lower expected wage, but it also features a larger expected compensation for the optimistic agent (and for the trusty agent). This sets the stage for the main result of this section.

Proposition 12. *If the agent is optimistic (or trusty) and his beliefs satisfy* (9) *and* (15)*, the principal offers a contract that costs her a lower expected wage and grants the optimistic or trusty agent a larger expected compensation. In this region, the agent's bias is* socially desirable*.*

In the proof of the Proposition we provide a formal argument to show that the region where optimism is socially desirable corresponds (in shape) to the one in Figure 7 below and that it always exists.[18]

We now carry on a similar comparison for the PED contracts.

Proposition 13. *Let $\tilde{E}(\cdot)$ denote the biased expectations of the agent. Given the baseline contract (BPE) $\{w^*_{ts}, c^*_{ts}\}$, the PED-DL $\{\hat{w}_{ts}, \hat{c}_{ts}\}$ and the PED-NDL $\{\hat{w}'_{ts}, \hat{c}'_{ts}\}$ contracts, the following are true:*

[18]The magnitude of the area, however, is purely indicative.

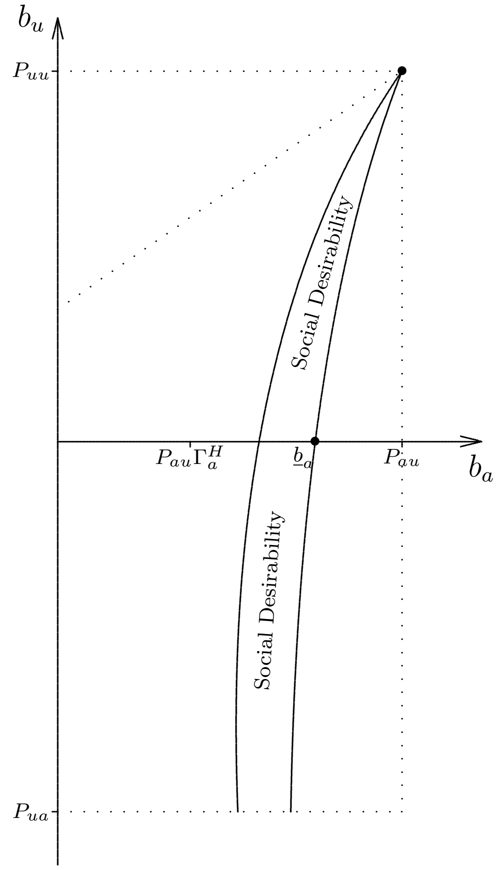

FIGURE 7. The area inside the two curves features a contract with a higher expected compensation and a lower expected wage compared to the benchmark contract assigned to an unbiased agent. Inside this area, the presence of an optimistic or trusty agent is socially optimal. In the proof of Proposition 12 we provide a formal argument to show that this area always exists and it is shaped as is displayed here. The value of $\underline{b}_a$ is also derived in the proof. This specific graph was obtained for $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. Its shape, however generalises to all feasible parameter values. The size and position of the area where the APE contract is offered is unaltered.

(i) $E(\hat{w}'_{ts}) \leq E(\hat{w}_{ts}) < E(w^*_{ts})$ *whenever the PED contracts are optimal.*
(ii) $\tilde{E}(\hat{c}_{ts}) = \tilde{E}(\hat{c}'_{ts}) > \tilde{E}(c^*_{ts})$ *always.*
(iii) $E(\hat{c}_{ts}) > E(c^*_{ts})$ *whenever* (14) *fails.*

Point (i) states, once again, that whenever PED contracts are assigned, they must be optimal. There is a difference, however, compared to the case of the APE. It is trivial to observe that $E(\hat{w}'_{ts}) < E(\hat{w}_{ts})$ whenever (11) strictly holds, since the PED contracts feature the same payments but for $\hat{w}'_{ua} < \hat{w}_{ua}$. As a matter of fact, the principal would always like to set up the PED-NDL contract rather than the PED-DL one. The former however, may not be feasible under some parameter conditions, like we show in Figure 6. Point (ii) is due to the wrong believed direction of correlation between signals by the agent. Given Lemma 10, we know that the PED contracts feature the largest possible compensations among the contracts set up for the pessimistic agent and for the skeptical agent, while featuring zero compensation in the agreement states. The bias of the agent is, however, enough for him to believe that his expected compensation is higher under a PED contract than under the baseline one. This, once again, connects to the idea of exploitation, where the principal takes advantage of the bias of the agent and connects to point (iii). Point (iii) follows the same intuition behind point (iv) of Proposition 11 but provides even more interesting insights summarized in the following Proposition.

Proposition 14. *A PED-NDL contract is never socially desirable. A PED-DL contract is socially desirable whenever it is optimal, feasible and* (14) *fails.*

Proposition 14 states a very controversial result on the PED-NDL contract. On the one hand, the PED-NDL contract features no deadweight loss. On the other hand, however, with a PED-NDL contract the principal takes so much advantage of the agent's biased beliefs that the agent never gains from switching from a BPE to a PED-NDL contract.

On the positive side, however, social desirability may take place when the PED-DL contract is assigned. When the principal is not capable of eliminating the deadweight loss, her taking advantage of the agent's bias may put the latter in a better position compared to the baseline contract. To see that this is possible, consider Figure 8 below where we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$. Figure 8 replicates the shapes of Figure 5. The shaded area may be larger for different parameter configurations. Figure 9 represents a case where a Pareto improvement is possible also for a pessimistic (and not skeptical) agent.

To conclude this section we present a result on the deadweight loss of each contract.

Proposition 15. *The BPE contract features the highest deadweight loss. That is*

$$\sum_{ts}(w_{ts}^{\dagger} - c_{ts}^{\dagger})\gamma_{ts}^{H} < \sum_{ts}(w_{ts}^{*} - c_{ts}^{*})\gamma_{ts}^{H}$$
$$\sum_{ts}(\hat{w}_{ts} - \hat{c}_{ts})\gamma_{ts}^{H} < \sum_{ts}(w_{ts}^{*} - c_{ts}^{*})\gamma_{ts}^{H}.$$

Of course, the contract with the lowest deadweight loss is the PED-NDL contract since it features none. The PED-DL contract features very little deadweight loss whenever the agent has a bias close to values that have (11) binding. Hence, which contract between PED-DL and APE features a smaller deadweight loss is not a trivial question to answer. However, since these contracts are never assigned to the same type of agent, it is also a relatively uninteresting question to look at.

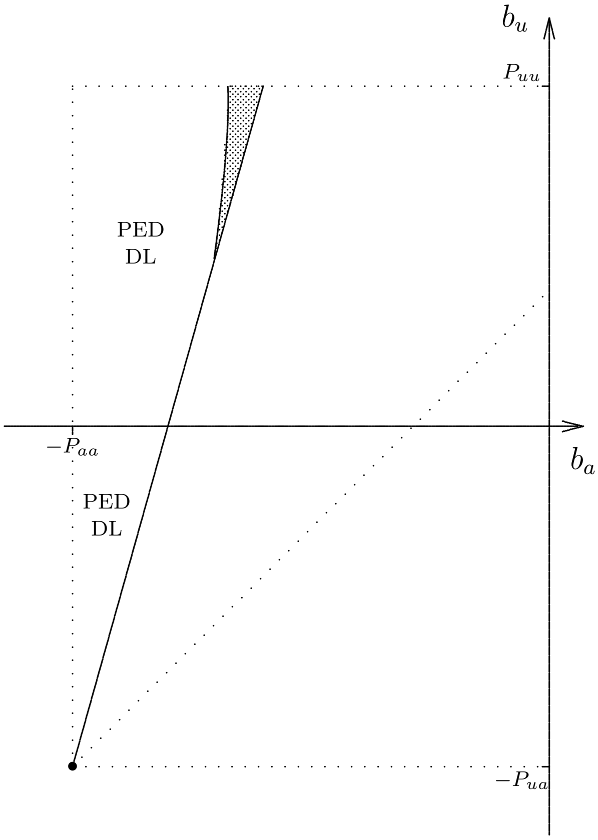

FIGURE 8. The shaded area between the two curves features a PED-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of a skeptical agent is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

Proposition 15 shows how the presence of a biased agent may be good for society (and mostly the principal) in terms of wasting less resources compared to the baseline case.

8. Concluding Remarks

This paper focuses on understanding the impact of workers' behavioral biases on subjective performance evaluation contracts. We have shown that while the benchmark contract assigned to an unbiased worker (Baseline Performance Evaluation contract) may also be assigned to biased workers under some conditions, three new contracts may arise in equilibrium. All of them share a main driving force: the principal tries to take advantage of the bias of the agent by altering the compensation levels compared to the benchmark case. If an agent is particularly optimistic about his performance, the principal may assign him an Agent's Performance Evaluation (APE) contract where the agent's compensation depends on the his signal and features a particularly high value in the case of a performance deemed acceptable by both agent and principal. If the agent

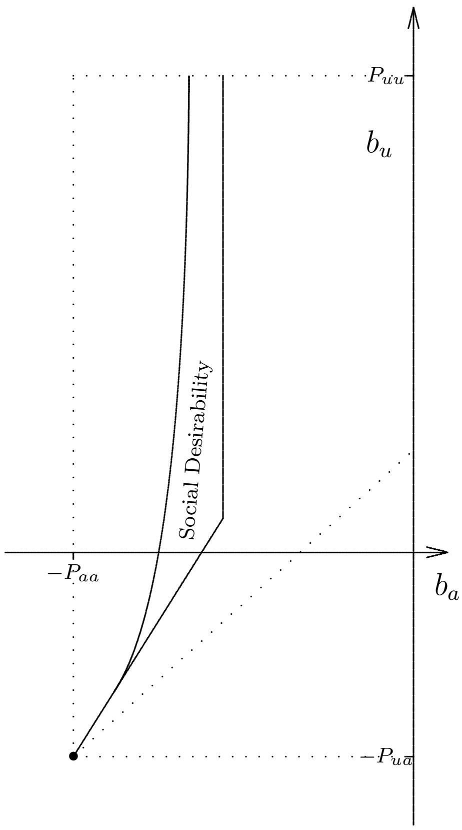

FIGURE 9. The area between the curve and the straight lines on the left features a PED-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of a pessimistic or skeptical agent is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.55, 0.7, 0.5)$.

is pessimistic enough to believe that signals are negatively correlated, the principal takes advantage of this bias by offering him a Performance Evaluation Disagreement (PED) contract, where no compensation is offered whenever the two parties agree on the performance evaluation. Under some conditions, this contract features no deadweight loss. These contracts differ from the BPE one in (i) how they use information, (ii) how they exploit the agent's bias, (iii) how much conflict and deadweight loss they feature and (iv) on their social desirability.

The standard BPE contract offers the agent a compensation that is independent of his own signal. If the principal wants to take advantage of the agent's bias, she ought to tie the compensation to the agent's signal. In this way she can promise him higher compensation in states he deems more probable and less compensation in states he deems less probable. In all the new contracts derived the agent's compensation depends both on T and on S. While the APE, however, features a wage payment that depends only on S, the PED contracts feature both compensation and wages depending on both signals. Hence, in PED contracts, no informational source is wasted at the contracting stage.

Using the mechanism just described, all contracts promise the agent an higher *perceived* expected compensation compared to the BPE one. In other words, the agent is convinced he will obtain more (in expectation) compared to the BPE contract, but whether that is actually the case it depends on how biased he is. While the motive that drives the principal to take advantage of the bias of the worker is purely to decrease the expected wage payment, her manipulation of the BPE contract may lead to a higher actual expected compensation for the agent by lowering the deadweight loss emerging from conflict. When this happens both parties obtain a larger welfare–a lower expected wage cost for the principal and a higher actual expected wage for the agent–than they would if the agent was unbiased. Hence, under such situations, the bias of the agent is socially desirable.

We have shown how a Pareto improvement happens for an "averagely" optimistic (or trusty) agent, who is optimistic enough for the principal to have some freedom to manipulate the contract in order to lower the principal's expected wage cost but without making the agent worse off. We have also shown how such an improvement may happen if the pessimistic (or skeptical) agent believes signals to be negatively correlated. Similarly, the agent cannot be too pessimistic otherwise the principal fully exploits his bias and takes away all his surplus. Finally and interestingly, we have also shown how a contract that features no conflict can never lead to a Pareto improvement. The intuition behind this surprising result has to be found in the origin of the Pareto improvements. As described, the principal does not have as an objective to increase social welfare. His incentive to decrease conflict via the exploitation of the agent's bias, if not taken too far, can however still make society achieve a better outcome. It goes without saying that in order to erase the entire conflict and deadweight loss from the contract, the exploitation of the agents' pessimistic beliefs must be substantial. In fact, enough to lower the worker's expected compensation.

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Appendix A. Proofs

Proof of Lemma 1

To prove (i) simply notice that:

$$\begin{aligned}\gamma^j_{ts} &= \lambda^j \gamma^G_{ts} + (1-\lambda^j)\gamma^B_{ts} \\ &= \lambda^j P_{ts}\gamma^G_t + (1-\lambda^j)P_{ts}\gamma^B_t \\ &= P_{ts}\left[\lambda^j\gamma^G_t + (1-\lambda^j)\gamma^B_t\right] = P_{ts}\Gamma^j_t.\end{aligned}$$

Now use (i) to substitute for the γ^J_{ts} in the equation of Assumption 2 to obtain:

$$\begin{aligned}&\gamma^j_{aa}\gamma^j_{uu} - \gamma^j_{au}\gamma^j_{ua} > 0 \\ &P_{aa}P_{uu}\Gamma^j_a\Gamma^j_u - P_{au}P_{ua}\Gamma^j_a\Gamma^j_u > 0 \\ &P_{aa}P_{uu} - P_{au}P_{ua} > 0 \qquad (16)\end{aligned}$$

by positivity of $\Gamma^j_a\Gamma^j_u$. Finally to prove (iii), notice that $P_{aa} = 1-P_{au}$ and $P_{ua} = 1-P_{uu}$. Substitute for the latter in (16) to obtain:

$$\begin{aligned}&(1-P_{au})P_{uu} - P_{au}(1-P_{uu}) > 0 \\ &P_{uu} - P_{au} > 0\end{aligned}$$

Similarly, substitute for $P_{au} = 1 - P_{aa}$ and $P_{uu} = 1 - P_{ua}$ in (16) to obtain that $P_{aa} - P_{ua} > 0$

Finally to prove (iv) note that

$$\begin{aligned}\Delta\Gamma_t &= \Gamma^H_t - \Gamma^L_t \\ &= \lambda^H\gamma^G_t + (1-\lambda^H)\gamma^B_t - \left[\lambda^L\gamma^G_t + (1-\lambda^L)\gamma^B_t\right] \\ &= \left(\lambda^H - \lambda^L\right)\left(\gamma^G_t - \gamma^B_t\right).\end{aligned}$$

Therefore

$$\begin{aligned}\Delta\Gamma_a + \Delta\Gamma_u &= \left(\lambda^H - \lambda^L\right)\left(\gamma^G_a - \gamma^B_a\right) - \left(\lambda^H - \lambda^L\right)\left(\gamma^G_u - \gamma^B_u\right) \\ &= \left(\lambda^H - \lambda^L\right)\left[(\gamma^G_a - \gamma^G_u) - (\gamma^B_u - \gamma^B_a)\right] \\ &= \left(\lambda^H - \lambda^L\right)\left[(\gamma^G_a - \gamma^G_u) - (1 - \gamma^G_u - 1 + \gamma^G_a)\right] \\ &= \left(\lambda^H - \lambda^L\right)\left[(\gamma^G_a - \gamma^G_u) - (\gamma^G_a - \gamma^G_u)\right] \\ &= 0.\end{aligned}$$

Proof of Lemma 2

Simple checking yields:

$$\tilde{\gamma}^j_{aa}\tilde{\gamma}^j_{uu} - \tilde{\gamma}^j_{au}\tilde{\gamma}^j_{ua} = (\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua})\Gamma^j_a\Gamma^j_u$$

which is positive when

$$\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua} = \tilde{P}_{aa}(1 - \tilde{P}_{ua}) - (1 - \tilde{P}_{aa})\tilde{P}_{ua}$$
$$= \tilde{P}_{aa} - \tilde{P}_{aa}\tilde{P}_{ua} - \tilde{P}_{ua} + \tilde{P}_{aa}\tilde{P}_{ua} = P_{aa} - P_{ua} + b_a - b_u > 0.$$

Since by Lemma 1 $P_{aa} > P_{ua}$, the latter inequality is always positive for $b_a \geq b_u$. For values of $b_u > b_a$, it yields condition (2).

Proof of Lemmas 3 and 4

Rearranging the two (TR_P) constraints:

$$(w_{ua} - w_{aa}) \geq (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}}$$

$$\Rightarrow (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}}. \tag{17}$$

Given Assumption 2, either all the brackets in (17) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 3.

For Lemmas 4 follow the same steps with the (TR_A) constraints to obtain:

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} \leq (c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}. \tag{18}$$

When the agent believes signals are positively correlated, i.e., $\tilde{\gamma}^H_{aa}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{\gamma}^H_{ua} > 0$, we have:

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} < \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}.$$

Given this last inequality, either all the brackets in (18) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 4.

Proof of Proposition 1

Suppose not, then $w_{ts} = c_{ts}$ for all t and s. Given Lemma 3 and 4, then we have:

$$c_{uu} \geq c_{ua} \geq c_{aa} \geq c_{au} \geq c_{uu}$$

Where the first and third inequalities follow from Lemma 4 and the second and fourth follow from Lemma 3. Obviously for all inequalities to hold together we need

$$c_{uu} = c_{ua} = c_{aa} = c_{au}.$$

This implies that $\tilde{E}(c_{ts}|\lambda^H) = \tilde{E}(c_{ts}|\lambda^L)$ since the agent compensation is completely independent from the realization of t and s. This of course violates the (IC) constraint since

$$\tilde{E}(c_{ts}|\lambda^H) - V(\lambda^H) < \tilde{E}(c_{ts}|\lambda^L) - V(\lambda^L).$$

Proof of Lemma 5

When the agent believes signals are negatively correlated, i.e., $\tilde{\gamma}^H_{aa}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{\gamma}^H_{ua} < 0$, we have:

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} > \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}.$$

That is the TR_A becomes

$$(c_{ua} - c_{uu})\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq (c_{au} - c_{aa}) \leq (c_{ua} - c_{uu})\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}.$$

Where either all brackets are 0 or $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$.

Reducing the problem to (6)

Lemma 6 below states that an agent believing that signals are positively correlated ought to be compensated in the "most positive" case, that is, when both principal and agent observe a signal reporting an acceptable performance. It also states that the agent obtains no compensation when the principal deems the performance unacceptable and the agent disagrees. Together with Lemma 8 below, Lemma 6 proves that the existence of a *deadweight loss* happens only when the principal deems the performance unacceptable, contrary to what the agent believes.

Lemma 6. *If the agent believes signals are positively correlated, i.e. ((2)) holds, then any optimal contract implementing high effort features $c_{aa} > c_{ua} = 0$.*

Proof. Define $\Delta\gamma_{ts} = \gamma^H_{ts} - \gamma^L_{ts}$ and $\Delta\tilde{\gamma}_{ts} = \tilde{\gamma}^H_{ts} - \tilde{\gamma}^L_{ts}$. First we prove that $\Delta\tilde{\gamma}_{as} > 0$ and $\Delta\tilde{\gamma}_{us} < 0$ for any $s \in \{a, u\}$ (it is easy to see that the same holds for $\Delta\gamma_{as}$ and $\Delta\gamma_{us}$). Notice that Assumption 1 is independent from Assumption 3. Therefore:

$$\begin{aligned}
\Delta\tilde{\gamma}_{ts} &= \tilde{\gamma}^H_{ts} - \tilde{\gamma}^L_{ts} \\
&= \lambda^H\tilde{\gamma}^G_{ts} + (1-\lambda^H)\tilde{\gamma}^B_{ts} - \lambda^L\tilde{\gamma}^G_{ts} - (1-\lambda^L)\tilde{\gamma}^B_{ts} \\
&= \lambda^H\tilde{P}_{ts}\gamma^G_t + (1-\lambda^H)\tilde{P}_{ts}\gamma^B_t - \lambda^L\tilde{P}_{ts}\gamma^G_t - (1-\lambda^L)\tilde{P}_{ts}\gamma^B_t \\
&= (\lambda^H - \lambda^L)\tilde{P}_{ts}(\gamma^G_t - \gamma^B_t),
\end{aligned}$$

which is positive at $t = a$ and negative otherwise.[19] Now we rewrite (IC) in the following way:

$$c_{aa}\Delta\tilde{\gamma}_{aa} + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} + c_{uu}\Delta\tilde{\gamma}_{uu} \geq \Delta V, \tag{19}$$

Recall that any optimal contract with truthful reporting for an agent who believes signals are positively correlated satisfies either case (i) or case (ii) of Lemma 4. Assume

[19]For future reference, this also proves that, as long as b_a and b_u are both positive, $\Delta\tilde{\gamma}_{ta} > \Delta\gamma_{ta}$ and $\Delta\tilde{\gamma}_{tu} < \Delta\gamma_{tu}$ for any t.

case (i) of Lemma holds 4 then (19) becomes:

$$c_{aa}\underbrace{(\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au})}_{>0}+c_{uu}\underbrace{(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}_{<0}\geq\Delta V.$$

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{aa} > 0$ to always hold. Assume now case (ii) of Lemma 4 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. If $c_{aa} > 0$, the Lemma is trivially proven. If $c_{au} \geq 0$, case (ii) implies $c_{aa} > c_{au} \geq 0$. This proves the first part of Lemma 6.

To prove the second part of Lemma 6, we suppose it is false, i.e., at optimum the contract features $c_{ua} > 0$, and we prove that there exists a profitable deviation from it, which contradicts its optimality. First of all, from Lemma 4 we know that $c_{uu} \geq c_{ua}$ and also $c_{aa} \geq c_{au}$. The proof now depends on whether $c_{au} > 0$ or $c_{au} = 0$.

Let $c_{au} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From (19) above we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{au} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way the overall change in the LHS of the (IC) is given by:

$$\begin{aligned}
&-\epsilon\left(\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au}+\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu}\right)\\
&\quad=-\epsilon\left(\tilde{P}_{aa}\Delta\Gamma_a+\tilde{P}_{au}\Delta\Gamma_a+\tilde{P}_{ua}\Delta\Gamma_u+\tilde{P}_{uu}\Delta\Gamma_u\right)\\
&\quad=-\epsilon\left(\Delta\Gamma_a+\Delta\Gamma_u\right)=-\epsilon\left(\Delta\Gamma_a-\Delta\Gamma_a\right)=0
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, then the principal can decrease also w_{ua} and w_{aa} by the same amount. This does not violate the the relevant (LL) and holds their difference constant. Hence, it does not violate any of the (TR_P) constraints. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{ua} > 0$ and $c_{au} > 0$ cannot be the solution to the problem.

Let now, instead, the optimal contract feature $c_{au} = 0$ and define $\Delta c_u = c_{uu} - c_{ua}$. We divide the proof for this case in three steps.

Step 1

When $c_{au} = 0$, the (TR_A) imply:

$$\Delta c_u\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}\leq c_{aa}\leq\Delta c_u\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}},\tag{20}$$

where, since we are in case (ii) of Lemma 4 either only one of the two inequalities holds as equality, or none. Suppose none of the two is strict, or the second one is, the principal can decrease both c_{uu} and c_{ua} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})$. He can then decrease c_{aa} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{aa}}$ bringing the LHS of the (IC) back to its original value. Clearly for some ϵ, this deviation can be done until the first inequality in (20) binds. Finally, to see that this is optimal for the principal, notice that according to the (LL) constraints, she can now decrease w_{ua} up to ϵ and w_{aa} up to δ. By decreasing both by $\min\{\epsilon,\delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected while the objective function decreases. This implies that at optimum if $c_{au}=0$, the first inequality of (20) binds.

Step 2

Given that $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$ must hold at optimum if $c_{au}=0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{au}=0$. Let her decrease c_{uu} by ϵ and c_{ua} by $\epsilon_0<\epsilon$. Then Δc_u has decreased by $(\epsilon-\epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$, the principal decreases c_{aa} by $(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$
\begin{aligned}
&-(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}\Delta\tilde{\gamma}_{aa} - \epsilon_0\Delta\tilde{\gamma}_{ua} - \epsilon\Delta\tilde{\gamma}_{uu} \\
&= -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{P}_{aa}\Gamma^H_a}\tilde{P}_{aa}\Delta\Gamma_a - \epsilon_0\tilde{P}_{ua}\Delta\Gamma_u - \epsilon\tilde{P}_{uu}\Delta\Gamma_u \\
&= -(\epsilon-\epsilon_0)\frac{\tilde{P}_{ua}\Gamma^H_u}{\Gamma^H_a}\Delta\Gamma_a + \epsilon_0\tilde{P}_{ua}\Delta\Gamma_a + \epsilon\tilde{P}_{uu}\Delta\Gamma_a \\
&= \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{uu} - \tilde{P}_{ua}\frac{\Gamma^H_u}{\Gamma^H_a}\right) + \epsilon_0\tilde{P}_{ua}\left(\frac{\Gamma^H_u}{\Gamma^H_a}+1\right)\right] \\
&= \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{uu} - \tilde{P}_{ua}\frac{1-\Gamma^H_a}{\Gamma^H_a}\right) + \epsilon_0\tilde{P}_{ua}\left(\frac{\Gamma^H_u}{\Gamma^H_a}+1\right)\right] \\
&= \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{uu}\Gamma^H_a - \tilde{P}_{ua} + \tilde{P}_{ua}\Gamma^H_a\right) + \epsilon_0\tilde{P}_{ua}\right] \\
&= \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\underbrace{(\tilde{P}_{uu}+\tilde{P}_{ua})}_{=1}\Gamma^H_a - \tilde{P}_{ua}\right) + \epsilon_0\tilde{P}_{ua}\right] \\
&= \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\Gamma^H_a - \tilde{P}_{ua}\right) + \epsilon_0\tilde{P}_{ua}\right],
\end{aligned}
$$

which is positive when:

$$\epsilon\left(\Gamma^H_a - \tilde{P}_{ua}\right) + \epsilon_0\tilde{P}_{ua} > 0.$$

If $\Gamma^H_a > \tilde{P}_{ua}$, the above is always true. If instead $\Gamma^H_a < \tilde{P}_{ua}$ then the principal has to choose $\epsilon \in \left\{\epsilon_0, \epsilon_0\frac{\tilde{P}_{ua}}{\tilde{P}_{ua}-\Gamma^H_a}\right\}$.

Step 3

To conclude, given the decreases in the c_{ts}, the principal can now decrease w_{ua} up to ϵ_0 and w_{aa} up to $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}$. By an argument similar to the one is Step 1, she can decrease both by the smallest of the two limits, decreasing the objective function. This provides the desired contradiction and hence a contract where $c_{ua} > 0$ and $c_{au} = 0$ cannot be the solution to the problem.

Finally, since a contract where $c_{ua} > 0$ and $c_{au} \geq 0$ cannot be a solution to the problem it follows that $c_{ua} = 0$. This concludes the proof of the Lemma. ∎

We can now move on to studying the principal's incentives. When $T = a$, clearly, she has an incentive not to reveal to the agent that she deems his performance acceptable, otherwise she has to pay him a premium. At optimum, this makes (TR^a_P) bind.

Lemma 7. *If the agent believes signals are positively correlated, i.e. ((2)) holds, then constraint (TR^a_P) always binds in any optimal contract implementing high effort.*

Proof. Of course in case (i) of Lemma 3 the Lemma is trivially proven. Assume now case (ii) of Lemma 3 holds and suppose (TR^a_P) is slack. If $w_{ua} = 0$, by Lemma 3 $w_{aa} = 0$ as well and case (ii) cannot happen. Now suppose $w_{ua} > 0$. We have $c_{ua} = 0$ from Lemma 6, and the principal can simply decrease w_{ua} until (TR^a_P) binds. This would relax (TR^u_P), not affect (LL_{ua}) and decrease the objective function. This proves the Lemma. ∎

The following Lemma allows us to write all w_{ts} as a function of the c_{ts}. It also shows, more formally, how the deadweight loss happens in the case of $T = u$ and $S = a$ only. That is, the case where the agent believes performance to be acceptable, disagreeing with the principal.

Lemma 8. *If the agent believes signals are positively correlated, i.e. ((2)) holds, then any optimal contract implementing high effort features:*

(i) $w_{aa} = c_{aa}$;
(ii) $w_{uu} = c_{uu}$;
(iii) $w_{au} = \max\{c_{au}, c_{uu}\}$;
(iv) $w_{ua} = c_{aa} + (\max\{c_{au}, c_{uu}\} - c_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$;

Proof. First of all notice that, by Lemma 7, $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}$. Hence, the principal's objective function in (3) can be rearranged as:

$$w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + \left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}}\right]\gamma^H_{ua} + w_{uu}\gamma^H_{uu}$$

and further as:

$$w_{aa}\left(\gamma^H_{aa} + \gamma^H_{ua}\right) + w_{au}\left(\gamma^H_{au} + \frac{\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}}\right) + w_{uu}\left(\gamma^H_{uu} - \frac{\gamma^H_{au}\gamma^H_{ua}}{\gamma^H_{aa}}\right)$$

where the last bracket is positive by Assumption 2. Furthermore, setting $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$ in (TR_P^u) we have

$$\left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}\right]\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H,$$

which is equivalent to

$$w_{uu} \leq w_{au}.$$

Hence, given the Lemmas so far, w_{aa}, w_{au}, and w_{uu} are only bound by $w_{uu} \leq w_{au}$ and the three corresponding (LL_{ts}). This implies that w_{aa}, w_{au}, and w_{uu} will be set to the lowest possible value. By Lemma 3 and in order to minimize the objective function, $w_{aa} = c_{aa}$, $w_{uu} = c_{uu}$ and $w_{au} = \max\{c_{au}, w_{uu}\}$, implying points (i), (ii) and (iii) of Lemma 8. Point (iv) follows by substitution. ∎

The next Lemma completes case (ii) of Lemma 4 by ranking c_{au} and c_{uu}. As expected, when the principal deems the performance acceptable, the agent may obtain a compensation premium even when he observes $S = u$.

Lemma 9. *If the agent believes signals are positively correlated, i.e. ((2)) holds, then any optimal contract implementing high effort features* $c_{au} \geq c_{uu}$.

Proof. Suppose not. Then $c_{uu} > c_{au} \geq 0$. By Lemma 6, $c_{ua} = 0$. Hence $c_{uu} > c_{ua}$, implying we are in case (ii) of Lemma 4 and $c_{aa} > c_{au}$. By Lemma 8 we then have $w_{uu} = w_{au} = c_{uu}$ and $w_{ua} = c_{aa} = w_{aa}$. This implies that c_{au} disappears from the objective function and from the constraints that affect the principal. She can, therefore, increase c_{au} and decrease the other levels of compensation (and therefore wage payments) in such a way that the rest of the constraints are still satisfied. This operation can be repeated until $c_{au} = c_{uu}$. Hence, the contradiction. ∎

Given this, we can further decrease the amount of binding constraints by proving the following:

Lemma 10. *If the agent believes signals are positively correlated, i.e. ((2)) holds, then constraint* (TR_A^u) *always binds in any optimal contract implementing high effort. Therefore:*

$$c_{uu} = \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}(c_{aa} - c_{au}).$$

Proof. Let $c_{uu} = 0$ then we are in case (i) of Lemma 4 and (TR_A^u) is trivially binding. Suppose now that $c_{uu} > 0$ and (TR_A^u) is not binding. We can then reduce c_{uu} until it binds. Given the proven Lemmas, the (TR_P) still hold, while (TR_A^a) and (IC) are relaxed by this change. To complete the proof we need to check whether a decrease in c_{uu} would decrease the objective function as well. By Lemmas 8 and 9, we can substitute for all wages in the objective function and find that the coefficient of c_{uu} becomes $\left(\gamma_{uu}^H - \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right)$, which is positive by Assumption 2. Hence, decreasing c_{uu} also decreases cost and it is therefore optimal for the principal to do so. This provides the desired contradiction and proves that (TR_A^u) always binds at optimum. ∎

This concludes the set of Lemmas yielding problem (6). Notice that, when plugging in value from Lemma 8 the objective function in (5), simplifies to (6) divided by $\gamma_{aa}^H \tilde{\gamma}_{uu}^H$. This is however irrelevant for the minimization problem and therefore omitted. Finally, to derive (7) and (8) , simply notice that (7) comes from the combination of Lemmas 9 and 10, while (8) derives from Lemma 4.

Optimism and Incentives. We can also prove the following two Lemmas to characterize the impact of optimism on the (IC) constraint. This allows us to present the rest of the results in a more intuitive way.

Lemma 11. *If the optimal contract implementing high effort features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, then optimism has no impact on the (IC).*

Proof. see the proof for Lemma 12. ∎

Lemma 11 simply states that if the agent's compensation is independent of the agent's signal, then the agent's optimism over their joint distribution has no effect on the (IC), and therefore on implementability of any level of effort.

Lemma 12. *If the optimal contract implementing high effort features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$, then optimism relaxes the (IC).*

Proof. The (IC)

$$\sum_{ts} c_{ts}(\tilde{\gamma}_{ts}^H - \tilde{\gamma}_{ts}^L) \geq \Delta V,$$

can be rewritten as

$$\sum_{ts} c_{ts}(\gamma_{ts}^H - \gamma_{ts}^L) + (c_{aa} - c_{au})(\Gamma_a^H - \Gamma_a^L)b_a + (c_{ua} - c_{uu})(\Gamma_u^H - \Gamma_u^L)b_u \geq \Delta V. \tag{21}$$

Note that $\Gamma_a^H > \Gamma_a^L$ and $\Gamma_u^H < \Gamma_u^L$. It follows directly from (21) that if the optimal contract features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, then optimism has no impact on the (IC). This proves Lemma 11. If the optimal contract features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$, then the second and third terms in the LHS of (21) are strictly positive and therefore optimism relaxes the IC. This proves Lemma 12. ∎

By Lemma 4, the agent knows that given what the principal observes, he obtains a premium when he reports $T = S$. A positive b_a (b_u) increase (decreases) the agent's belief of both signals to show a (u). This means that, given effort, an optimistic agent with beliefs satisfying (2) overestimates the chances of obtains the premium $c_{aa} - c_{au}$ and underestimates the ones of obtaining $c_{uu} - c_{ua}$. Since $T = a$ is most probable when he exerts high effort, the agent requires a lower incentive to exert λ^H. That is to say, exerting high effort is part of his "strategy" to increase the chance of $T = S = a$.

Proof of Proposition 2

The inequality in Proposition 2 follows from the comparisons of the slope of the (IC) with the slope of the iso-costs. This produces the following condition, that we simplify as follows.

$$\frac{\Delta\tilde{\gamma}_{au}-\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa}+\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}\leq\frac{\gamma^H_{aa}\gamma^H_{au}\tilde{\gamma}^H_{uu}+\gamma^H_{au}\gamma^H_{ua}\tilde{\gamma}^H_{uu}-\tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa}+\tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}{(\gamma^H_{aa})^2\tilde{\gamma}^H_{uu}+\gamma^H_{aa}\gamma^H_{ua}\tilde{\gamma}^H_{uu}+\tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa}-\tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}.$$

We start from simplifying the slope of the IC

$$\begin{aligned}
LHS &= \frac{\Delta\tilde{\gamma}_{au}-\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa}+\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}} = \frac{\tilde{\gamma}^H_{au}-\tilde{\gamma}^L_{au}-\tilde{\gamma}^H_{au}+\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^H_{aa}-\tilde{\gamma}^L_{aa}+\tilde{\gamma}^H_{au}-\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}}\\
&= \frac{\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}-\tilde{\gamma}^L_{au}}{\tilde{\gamma}^H_{aa}-\tilde{\gamma}^L_{aa}+\tilde{\gamma}^H_{au}-\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\tilde{\gamma}^L_{uu}} = \frac{\frac{\tilde{P}_{au}\tilde{P}_{uu}\Gamma^H_a\Gamma^L_u}{\tilde{P}_{uu}\Gamma^H_u}-\tilde{P}_{au}\Gamma^L_a}{\tilde{P}_{aa}\Delta\Gamma_a+\tilde{P}_{au}\Gamma^H_a\left(1-\frac{\Gamma^L_u}{\Gamma^H_u}\right)}\\
&= \frac{\tilde{P}_{au}\left(\Gamma^H_a\Gamma^L_u-\Gamma^L_a\Gamma^H_u\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma^H_u+\tilde{P}_{au}\Gamma^H_a\Delta\Gamma_u}
\end{aligned}$$

Notice that since $\Gamma^J_a+\Gamma^J_u=1$ for any $j=H,L$, then we can substitute for $\Gamma^H_u=1-\Gamma^H_a$ and $\Gamma^L_u=1-\Gamma^L_a$. Also, as already proven, $\Delta\Gamma_a=-\Delta\Gamma_u$. Hence we can further simplify the LHS:

$$\begin{aligned}
&= \frac{\tilde{P}_{au}\left(\Gamma^H_a\Gamma^L_u-\Gamma^L_a\Gamma^H_u\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma^H_u+\tilde{P}_{au}\Gamma^H_a\Delta\Gamma_u}\\
&= \frac{\tilde{P}_{au}\left(\Gamma^H_a(1-\Gamma^L_a)-\Gamma^L_a(1-\Gamma^H_a)\right)}{\tilde{P}_{aa}\Delta\Gamma_a(1-\Gamma^H_a)+\tilde{P}_{au}\Gamma^H_a(-\Delta\Gamma_a)}\\
&= \frac{\tilde{P}_{au}\Delta\Gamma_a}{\Delta\Gamma_a\left[\tilde{P}_{aa}(1-\Gamma^H_a)-\tilde{P}_{au}\Gamma^H_a\right]} = \frac{\tilde{P}_{au}}{\tilde{P}_{aa}-\Gamma^H_a(\tilde{P}_{aa}+\tilde{P}_{au})} = \frac{\tilde{P}_{au}}{\tilde{P}_{aa}-\Gamma^H_a}\\
&= \frac{P_{au}-b_a}{P_{aa}-\Gamma^H_a+b_a}.
\end{aligned}$$

The slope of the iso-costs, instead, is given by

$$\begin{aligned}
&\frac{\gamma^H_{aa}\gamma^H_{au}\tilde{\gamma}^H_{uu}+\gamma^H_{au}\gamma^H_{ua}\tilde{\gamma}^H_{uu}-\tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa}+\tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}{(\gamma^H_{aa})^2\tilde{\gamma}^H_{uu}+\gamma^H_{aa}\gamma^H_{ua}\tilde{\gamma}^H_{uu}+\tilde{\gamma}^H_{au}\gamma^H_{uu}\gamma^H_{aa}-\tilde{\gamma}^H_{au}\gamma^H_{au}\gamma^H_{ua}}\\
= \quad & \frac{(P_{uu}-b_u)\left(P_{aa}P_{au}\Gamma^H_a+P_{au}P_{ua}\Gamma^H_u\right)-(P_{au}-b_a)\Gamma^H_a\left(P_{aa}P_{uu}-P_{au}P_{ua}\right)}{(P_{uu}-b_u)\left(P_{aa}P_{aa}\Gamma^H_a+P_{aa}P_{ua}\Gamma^H_u\right)+(P_{au}-b_a)\Gamma^H_a\left(P_{aa}P_{uu}-P_{au}P_{ua}\right)}\\
= \quad & \frac{(P_{uu}-b_u)P_{au}\left(P_{aa}\Gamma^H_a+P_{ua}\Gamma^H_u\right)-(P_{au}-b_a)\Gamma^H_a\left(P_{aa}P_{uu}-P_{au}P_{ua}\right)}{(P_{uu}-b_u)P_{aa}\left(P_{aa}\Gamma^H_a+P_{ua}\Gamma^H_u\right)+(P_{au}-b_a)\Gamma^H_a\left(P_{aa}P_{uu}-P_{au}P_{ua}\right)}\\
= \quad & \frac{(P_{uu}-b_u)P_{au}Z-(P_{au}-b_a)W}{(P_{uu}-b_u)P_{aa}Z+(P_{au}-b_a)W},
\end{aligned}$$

where $Z = \left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)$ and $W = \Gamma_a^H\left(P_{aa}P_{uu} - P_{au}P_{ua}\right) = \Gamma_a^H(P_{aa} - P_{ua})$. Hence the inequality in Proposition 2 is equivalent to

$$\frac{P_{au} - b_a}{P_{aa} - \Gamma_a^H + b_a} \leq \frac{(P_{uu} - b_u)P_{au}Z - (P_{au} - b_a)W}{(P_{uu} - b_u)P_{aa}Z + (P_{au} - b_a)W},$$

or

$$\begin{aligned} & (P_{au} - b_a)(P_{uu} - b_u)P_{aa}Z + (P_{au} - b_a)^2 W \\ \leq \; & \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{uu} - b_u)P_{au}Z - \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{au} - b_a)W, \end{aligned}$$

or

$$\begin{aligned} & (P_{au} - b_a)^2 W + \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{au} - b_a)W \\ \leq \; & \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{uu} - b_u)P_{au}Z - (P_{au} - b_a)(P_{uu} - b_u)P_{aa}Z, \end{aligned}$$

or

$$(P_{au} - b_a)\left(P_{au} - b_a + P_{aa} - \Gamma_a^H + b_a\right)W \leq (P_{uu} - b_u)\left[\left(P_{aa} - \Gamma_a^H + b_a\right)P_{au} - (P_{au} - b_a)P_{aa}\right]Z,$$

or

$$(P_{au} - b_a)\left(P_{au} + P_{aa} - \Gamma_a^H\right)W \leq (P_{uu} - b_u)\left[b_a(P_{aa} + P_{au}) - P_{au}\Gamma_a^H\right]Z, \tag{22}$$

or

$$\frac{(P_{au} - b_a)\left(1 - \Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z} \leq P_{uu} - b_u,$$

or

$$b_u \leq P_{uu} - \frac{(P_{au} - b_a)\left(1 - \Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}. \tag{23}$$

If the second term on the RHS of (23) is non-negative, then the inequality places no new restriction on the space (b_a, b_u). However, if the second term on the RHS of (23) is negative, then the inequality places a new restriction on the space (b_a, b_u). Since $b_a \in (0, P_{au}]$ and $\Gamma_a^H \in (0, 1)$, the second term on the RHS of (23) is negative when $b_a \in (P_{au}\Gamma_a^H, P_{au})$.

Further, notice that if $b_a < P_{au}\Gamma_a^H$, inequality (22) cannot hold.

Proof of Proposition 3

The Proof is divided in two parts. First we show that when $b_a = b_u = 0$, the slope of the (IC) is never lower than the slope of the iso-costs. Then we derive the optimal contract for the unbiased agent.

Using the algebra presented in the proof of Proposition 2, consider the slope of the IC when the agent is unbiased:

$$\frac{P_{au}}{P_{aa} - \Gamma_a^H}$$

This implies that the (IC) is negatively sloped if and only if $\Gamma_a^H < P_{aa}$. First we assume $\Gamma_a^H < P_{aa}$ and show that the (24) always holds. Then we move to the case of $\Gamma_a^H > P_{aa}$.

The comparison between slopes then becomes:

$$\frac{P_{au}}{P_{aa}-\Gamma_a^H} > \frac{\gamma_{aa}^H\gamma_{au}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H - \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \tag{24}$$

Let $\Gamma_a^H < P_{aa}$. We now rearrange the RHS, which is less nicely simplified.

$$\begin{aligned} RHS &= \frac{\gamma_{aa}^H\gamma_{au}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H - \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \\ &= \frac{\gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \end{aligned}$$

Before going ahead, notice that this proves that in the case of an unbiased agent isocosts are always negatively slope. Carrying on we obtain

$$\begin{aligned} &\frac{\gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \\ &= \frac{P_{au}P_{ua}P_{uu}\Gamma_a^H\Gamma_u^H\Gamma_u^H + P_{au}P_{au}P_{ua}\Gamma_a^H\Gamma_a^H\Gamma_u^H}{P_{aa}P_{aa}P_{uu}\Gamma_a^H\Gamma_a^H\Gamma_u^H + P_{aa}P_{ua}P_{uu}\Gamma_a^H\Gamma_u^H\Gamma_u^H + P_{au}P_{uu}P_{aa}\Gamma_a^H\Gamma_a^H\Gamma_u^H - P_{au}P_{au}P_{ua}\Gamma_a^H\Gamma_a^H\Gamma_u^H} \\ &= \frac{P_{au}P_{ua}P_{uu}\Gamma_u^H + P_{au}P_{au}P_{ua}\Gamma_a^H}{P_{aa}P_{aa}P_{uu}\Gamma_a^H + P_{aa}P_{ua}P_{uu}\Gamma_u^H + P_{au}P_{uu}P_{aa}\Gamma_a^H - P_{au}P_{au}P_{ua}\Gamma_a^H} \\ &= \frac{P_{au}P_{ua}(P_{uu}\Gamma_u^H + P_{au}\Gamma_a^H)}{P_{aa}P_{uu}\Gamma_a^H(P_{aa}+P_{au}) + P_{ua}(P_{aa}P_{uu}\Gamma_u^H - P_{au}P_{au}\Gamma_a^H)} \\ &= \frac{P_{au}P_{ua}(P_{uu}(1-\Gamma_a^H) + P_{au}\Gamma_a^H)}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \\ &= \frac{P_{au}P_{ua}(P_{uu} - \Gamma_a^H(P_{uu}-P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \end{aligned}$$

This implies that comparing the slopes boils down to:

$$\begin{aligned} \frac{P_{au}}{P_{aa}-\Gamma_a^H} &> \frac{P_{au}P_{ua}(P_{uu} - \Gamma_a^H(P_{uu}-P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \\ \frac{1}{P_{aa}-\Gamma_a^H} &> \frac{P_{ua}(P_{uu} - \Gamma_a^H(P_{uu}-P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \end{aligned}$$

$$P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H) > (P_{aa}-\Gamma_a^H)P_{ua}(P_{uu} - \Gamma_a^H(P_{uu}-P_{ua}))$$

Recall that Lemma 1 showed $P_{aa} > P_{ua}$ and $P_{uu} > P_{au}$.

$$\begin{aligned} &P_{aa}P_{uu}\Gamma_a^H + P_{ua}P_{aa}P_{uu} - P_{ua}P_{aa}P_{uu}\Gamma_a^H - P_{ua}P_{au}P_{au}\Gamma_a^H \\ &> P_{ua}P_{aa}P_{uu} - P_{ua}P_{uu}\Gamma_a^H - P_{aa}P_{ua}\Gamma_a^H(P_{uu}-P_{ua}) + P_{ua}(\Gamma_A^H)^2(P_{uu}-P_{ua}) \end{aligned}$$

which, by simplifying and dividing by Γ_a^H on both sides, is equivalent to:

$$P_{aa}P_{uu} - P_{ua}P_{au}P_{au} > -P_{ua}P_{uu} + P_{aa}P_{ua}P_{ua} + P_{ua}P_{uu}\Gamma_A^H - P_{ua}P_{ua}\Gamma_A^H$$

$$P_{aa}P_{uu} - P_{ua}P_{au}^2 > -P_{ua}P_{uu} + P_{aa}P_{ua}^2 + P_{uu}P_{ua}\Gamma_a^H - P_{ua}^2\Gamma_a^H$$
$$P_{uu}(P_{aa} + P_{ua}) - P_{ua}\Gamma_a^H(P_{uu} - P_{ua}) - P_{ua}P_{au}^2 - P_{aa}P_{ua}^2 > 0$$

Now we substitute for $P_{uu} = 1 - P_{ua}$ and $P_{au} = 1 - P_{aa}$ and we get:

$$(1 - P_{ua})(P_{aa} + P_{ua}) - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua}(1 - P_{aa})^2 - P_{aa}P_{ua}^2 > 0$$
$$P_{aa} + P_{ua} - P_{aa}P_{ua} - P_{ua}^2 - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua} + 2P_{aa}P_{ua} - P_{ua}P_{aa}^2 - P_{aa}P_{ua}^2 > 0$$
$$P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + \underbrace{P_{ua}\Gamma_a^H(2P_{ua} - 1)}_{\mathbf{\Gamma}} > 0$$

Suppose first that $P_{ua} < \frac{1}{2}$, then $\mathbf{\Gamma} < 0$ and the LHS gets smaller the greater is Γ_a^H. Hence, to be sure the condition holds, we set $\Gamma_a^H = P_{aa}$, the highest possible value it can get. This yields $\mathbf{\Gamma} = 2P_{aa}P_{ua}^2 - P_{aa}P_{ua}$. Hence the condition becomes:

$$P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + 2P_{aa}P_{ua}^2 - P_{aa}P_{ua} > 0$$
$$P_{aa} + P_{aa}P_{ua}^2 - P_{aa}^2P_{ua} - P_{ua}^2 > 0 \tag{25}$$

Notice that if this holds for all $P_{aa} > P_{ua}$ then so will the condition for the case of $P_{ua} > \frac{1}{2}$. In that case, in fact, $\mathbf{\Gamma} > 0$, which means that the LHS would increase with Γ_a^H. Hence, to check it holds we set it to 0. This would set $\mathbf{\Gamma} = 0$ and yield a condition looser than (25).

To see that (25) always holds, notice that the derivative of the LHS with respect to P_{ua} is given by:

$$\frac{\partial LHS}{\partial P_{ua}} = 2P_{aa}P_{ua} - P_{aa}^2 - 2P_{ua} = 2P_{ua}(P_{aa} - 1) - P_{aa}^2$$

which is negative for all $P_{aa} < 1$. Hence, the condition is monotonically decreasing in P_{ua}. We therefore check for the maximum value of P_{ua}, which in this case is $\frac{1}{2}$. At this value, condition (25) becomes simply

$$-2P_{aa}^2 + 5P_{aa} - 1 > 0$$

By Lemma 1 P_{aa} must be *strictly* larger than P_{ua}. The second order equation above always holds for $P_{aa} \in [\frac{1}{2}, 1]$.

We are now left to show that when the (IC) is positively sloped the equilibrium contract coincides with the one presented in the Result. To do this, let $\Gamma_a^H > P_{aa}$. Notice that we can rearrange the intercept to obtain:

$$\frac{\Delta V}{(P_{aa} - \Gamma_a^H)\frac{\Delta\Gamma_a}{\Gamma_u^H}}$$

which is clearly negative when $\Gamma_a^H > P_{aa}$. Furthermore, given $\Gamma_a^H > P_{aa}$ the slope of the IC is greater than 1 as long as

$$\frac{P_{au}}{\Gamma_a^H - P_{aa}} > 1,$$

or

$$P_{au} > \Gamma_a^H - P_{aa},$$

or

$$P_{aa} + P_{au} = 1 > \Gamma_a^H,$$

which is true since $\Gamma_a^H \in (0,1)$. This result together with the fact that the iso-cost is always negatively sloped for an unbiased agent imply that the optimal contract for an unbiased agent with an IC with a positive slope is at $c_{aa} = c_{au}$. To see this, notice that the positively sloped (IC) and the 45° line cross only once in (c_{au}, c_{aa}) space, that will the point of optimal contracting.

Now we move to deriving the optimal contract. Consider problem (6) with $b_a = b_u = 0$. Given the proof so far, we can disregard (7) and let (8) and the (IC) bind. We are then left with a system of two equations in two variables. The solution is trivially obtain by substitution. Simply start from $c_{aa} = c_{au}$ and substitute it in the (IC) to obtain

$$c_{aa}(\Delta\gamma_{aa} + \Delta\gamma_{au}) = \Delta V$$

which yields

$$c_{aa}(\Delta\Gamma_a P_{aa} + \Delta\Gamma_a P_{au}) = \Delta V$$

and

$$c_{aa}(\Delta\Gamma_a) = \Delta V \Rightarrow c_{aa} = \frac{\Delta V}{\Delta\Gamma_a} = c_{au}.$$

Wages are obtained by substituting the compensation values into the wages of Lemma 8. Notice that

$$\begin{aligned} w_{ua} =& \frac{\Delta V}{\Delta\Gamma_a}\left(1 + \frac{\gamma_{au}^H}{\gamma_{aa}^H}\right) = \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{\gamma_{aa}^H + \gamma_{au}^H}{\gamma_{aa}^H}\right) \\ =& \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{P_{aa}\Gamma_a^H + P_{au}\Gamma_a^H}{P_{aa}\Gamma_a^H}\right) = \frac{\Delta V}{\Delta\Gamma_a}\left(\frac{1}{P_{aa}}\right) \end{aligned}$$

Proof of Proposition 4

To see that the contract completely resembles the baseline one, simply notice that the two problems are solved in the exact same way, and that

$$c'_{aa} = \frac{\Delta V}{\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au}} = \frac{\Delta V}{\Delta\Gamma_a(\tilde{P}_{aa} + \tilde{P}_{au})} = \frac{\Delta V}{\Delta\Gamma_a}$$

Proof of Proposition 5

The proof of the proposition is divided in four parts. First we show that when either or both the (IC) and the iso-costs are positively sloped, the optimal contract is the standard one. Then we derive conditions for this case to happen. Third, we prove that condition (9) implies all the conditions derived below as well as (2) — and hence all the latter can be omitted from a graphical analysis — and we identify the shape of the area where the APE contract is set up (i.e. we provide an explanation to the shape of

Figure 2). Finally, we move to deriving the values of wages and compensations of the APE contract.

Part 1. First of all, notice from (6) that an increase of c_{aa} always increases the expected cost of implementing high effort. The effect of an increase of c_{au}, however, is not straightforward. If it is positive, then iso-costs are negatively sloped in (c_{au}, c_{aa}) space and costs decrease towards the origin. If it is negative, then iso-costs are positively sloped and costs decrease towards the bottom right of the graph.

Suppose the latter is true. Since iso-costs are positively sloped in (c_{au}, c_{aa}) space, optimal contracts lie at point Y of Figure 1. Notice, however, that a further check is needed here. Suppose the iso-costs are positively sloped. If their slope is larger than 1, then they are steeper than the locus of points where $c_{aa} = c_{au}$. Hence, for any given $c_{aa} = c_{au} = c$, there would always exists a $c' > c$ lying on an iso-costs further to the right of Figure 1 satisfying all constraints and lowering costs. Hence, an optimal contract would feature $c_{aa} = c_{au} = c \to \infty$. In order to check that this cannot happen, we study the value of the slope of the iso-costs when the latter is positive. From the algebra in the proof of Proposition 2 we can get this value as:

$$\frac{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) - (P_{uu} - b_u)P_{au}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) + (P_{uu} - b_u)P_{aa}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}$$

which is trivially never larger than 1. Hence in equilibrium the baseline contract is set up.

Now suppose that the (IC) is positively sloped. This implies that it requires c_{aa} to be smaller than c_{au} times a positive number. First of all, notice from the (IC) that when it is positively sloped, its intercept is negative. Further, its slope is now given by

$$\frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}{-\Delta\tilde{\gamma}_{aa} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}$$

which is obviously larger than 1. Hence, the set of constraint compatible contracts becomes the one highlighted in Figure 10.

Regardless of whether the iso-costs are positively or negatively sloped, the optimal contract lies at point Y in the graph and replicate the standard contract.

Part 2. As already discussed, the slope of the (IC) is negative as long as $b_a \geq \Gamma_a^H - P_{aa}$. The condition for the slope of the iso-cost to be negative, instead, can be derived as follows.

Consider the slope derived in the proof of Proposition 2 again, this time without looking at its absolute value (i.e. we keep the minus in the front).

$$-\frac{(P_{uu} - b_u)P_{au}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) - (P_{au} - b_a)\Gamma_a^H\left(P_{aa} - P_{ua}\right)}{(P_{uu} - b_u)P_{aa}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) + (P_{au} - b_a)\Gamma_a^H\left(P_{aa} - P_{ua}\right)}$$

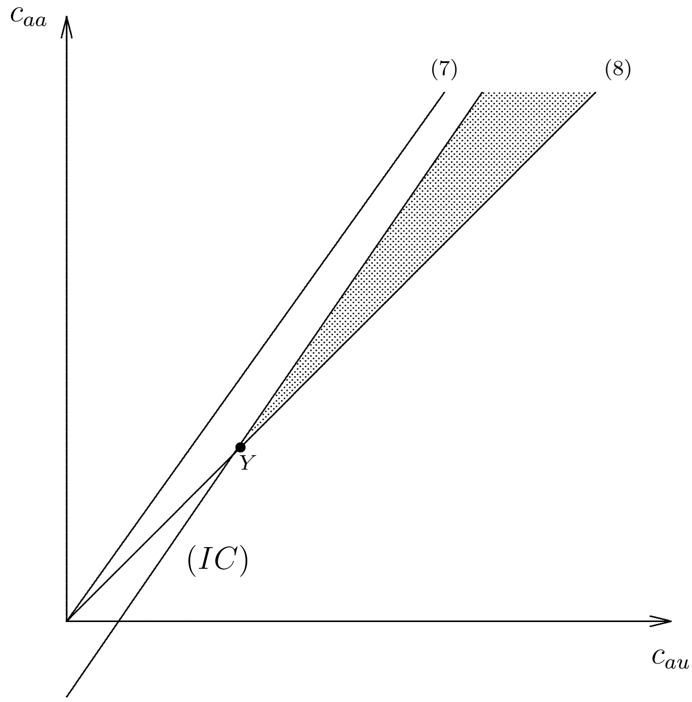

FIGURE 10. The shaded area represents the set of contracts satisfying all the constraints of the minimisation problem when the agent believes that signals are positively correlated and the (IC) is positively sloped.

It is easy to see then, that the iso-costs are negatively sloped when the numerator of the above is positive. This happens when:

$$(P_{uu} - b_u)P_{au}\underbrace{\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)}_{Z} - (P_{au} - b_a)\underbrace{\Gamma_a^H\left(P_{aa} - P_{ua}\right)}_{W} > 0$$

which yields the condition:

$$b_u < P_{uu} - \frac{(P_{au} - b_a)W}{P_{au}Z} \tag{26}$$

Part 3. In this part of the proof we show how, for $b_a \in \left[P_{au}\Gamma_a^H, P_{au}\right]$, condition (9) implies the negativity of the slope of the (IC), condition (2) and (26). We also show how the area it delimits has a concave shape in (b_a, b_u) space and how it always lies in the interval $(P_{au}\Gamma_a, P_{au})$ on b_a.

First of all, notice that the (IC) is negatively sloped if

$$b_a \geq \Gamma_a^H - P_{aa} = \Gamma_a^H - 1 + P_{au}$$

and that

$$\Gamma_a^H - 1 + P_{au} < P_{au}\Gamma_a^H \;\Rightarrow\; P_{au}(1 - \Gamma_a^H) < 1 - \Gamma_a^H.$$

Hence, when $b_a > P_{au}\Gamma_a^H$ (which is necessary for (9) to have meaning) the (IC) is negatively sloped.

We now compare (9) to (2). First notice that (2) is linear and rearrange it as

$$b_u \leq P_{aa} + b_a - P_{ua}.$$

Given the possible values of b_a, the value of the RHS goes from $P_{aa} + P_{au}\Gamma_a^H - P_{au}$ to $1 - P_{au} = P_{uu}$. Its derivative in b_a is obviously 1. Similarly, we can evaluate the RHS of (9) at $b_a = P_{au}$ to see that it is simply P_{uu}. This means that the two conditions coincide at $b_a = P_{au}$. Now notice that as $b_a \to P_{au}\Gamma_a^H$ the RHS of condition (9) goes to 0 (since the second term explodes and eventually reaches P_{uu}). We therefore have that condition (2) lies above (9) at the two boundaries for the feasible interval of b_a. We are left to check that the two stay this way over the entire interval. To see this, we study the derivative of the RHS of (9) and show that it is always positive and larger than 1, i.e. larger than the derivative of the RHS of (2). This ensures that the two curves cross only once.

$$\begin{aligned}&\frac{\partial}{\partial b_a}\left[P_{uu} - \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}\right]\\ &= - \frac{-(1-\Gamma_a^H)W\left(b_a - P_{au}\Gamma_a^H\right)Z - Z\left((P_{au}-b_a)\left(1-\Gamma_a^H\right)W\right)}{\left(\left(b_a - P_{au}\Gamma_a^H\right)Z\right)^2}\\ &= \frac{P_{au}(1-\Gamma_a^H)^2 WZ}{\left[\left(b_a - P_{au}\Gamma_a^H\right)Z\right]^2}\end{aligned}$$

which is always positive. To see that it is larger than 1 we calculate:

$$\frac{P_{au}(1-\Gamma_a^H)^2 WZ}{\left[\left(b_a - P_{au}\Gamma_a^H\right)Z\right]^2} > 1$$

which yields

$$P_{au}(1-\Gamma_A^H)^2 W - b_a^2 Z - P_{au}^2(\Gamma_A^H)^2 Z + 2P_{au}b_a\Gamma_a^H Z > 0.$$

The study of this inequality is not trivial. Consider first the derivative of the LHS with respect to b_a. It yields $P_{au}\Gamma_a^H - b_a$ which is always negative. Hence, if the condition holds at the lowest feasible value of b_a, it holds for all values of b_a. To see that this is the case, notice that as $b_a \to P_{au}\Gamma_a^H$ the LHS of the inequality above converges to:

$$P_{au}(1-\Gamma_A^H)^2 W - P_{au}^2(\Gamma_A^H)^2 Z - P_{au}^2(\Gamma_A^H)^2 Z + 2P_{au}^2(\Gamma_A^H)^2 Z = P_{au}(1-\Gamma_A^H)^2 W > 0.$$

Hence the slope of the RHS of (9) is always larger than the one of (2). This implies that the two cross only once and that (9) is always tighter than (2).

Comparing (9) with (26) is much simpler. It is enough for the RHS of (26) to be larger than (9). This comparison corresponds to comparing the second terms of the RHS of each inequality. Condition (26) is looser if

$$\frac{(P_{au}-b_a)W}{P_{au}Z} \geq \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a - P_{au}\Gamma_a^H\right)Z}$$

which corresponds to

$$P_{au}(1-\Gamma_a^H) \geq b_a - P_{au}\Gamma_a^H \;\Rightarrow\; P_{au} \geq b_a$$

which is always true.

To conclude this part of the proof we show that the RHS of (9) is concave in b_a. To see this consider the first derivative above,

$$\left[\frac{P_{au}(1-\Gamma_a^H)^2 WZ}{[(b_a - P_{au}\Gamma_a^H)\,Z]^2}\right],$$

and notice that it is decreasing in b_a. Hence, (9) identifies a concave area.[20] To see that its lower bound is always larger than $P_{au}\Gamma_a^H$ simply substitute $b_u = 0$ in the condition to obtain

$$0 \leq P_{uu} - \frac{(P_{au} - b_a)\left(1-\Gamma_a^H\right)W}{(b_a - P_{au}\Gamma_a^H)\,Z}$$

which is equivalent to

$$b_a \geq P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}$$

To prove our claim we then show that

$$\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z} > \Gamma_a^H.$$

With simple algebra it is easy to see that this condition boils down to $\Gamma_a^H \leq 1$, which is always true.

This concludes this part and proves that the area identified by the feasible values of b_a and condition (9) always features the APE contract. Its shape, furthermore, always resembles the representation in Figure 2.

Part 4. Given all the above and Proposition 2 we finally solve problem (6) by setting (7) binding together with the (IC). This yields the following system in two equations:

$$c_{aa}\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + c_{au}\left(\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) = \Delta V$$

$$c_{aa} = \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)c_{au}$$

[20] Recall that the derivative is of the entire RHS not only of the second term.

from which we obtain:

$$
\begin{aligned}
c_{au} &= \frac{\Delta V}{\left(1+\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right)\left(\Delta\tilde{\gamma}_{aa}+\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}\right)+\Delta\tilde{\gamma}_{au}-\frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{uu}}\Delta\tilde{\gamma}_{uu}} \\
&= \frac{\Delta V}{\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au}+\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{uu}} \\
&= \frac{\Delta V}{\Delta\Gamma_a\tilde{P}_{aa}+\Delta\Gamma_a\tilde{P}_{au}+\frac{\tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a}\Delta\Gamma_a\tilde{P}_{aa}-\Delta\Gamma_a\tilde{P}_{uu}} \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{1}{1+\frac{\tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a}\tilde{P}_{aa}-\tilde{P}_{uu}} \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a+\tilde{P}_{uu}(1-\Gamma^H_a)\tilde{P}_{aa}-\tilde{P}_{uu}\tilde{P}_{au}\Gamma^H_a} \\
&= c^*_{aa}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma^H_a)}
\end{aligned}
$$

To conclude the proof, we obtain $c_{aa} = \left(1+\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right)c_{au}$ from the above discussion, and $c_{au} = c_{uu}$ from Lemma 10.

Reducing the problem to (10)

Lemma 13 below studies the effect of optimism on the (IC) in this case. Intuitively, given Lemma 5, the agent now overestimates the chances of obtaining premium $c_{ua}-c_{uu}$, and underestimates the ones of obtaining $c_{au}-c_{aa}$. Hence, his incentive to exert λ^L is higher, since $T = u$ is more probable under low effort, and the (IC) tightens.

Lemma 13. *If the optimal contract implementing high effort features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then optimism tightens the (IC).*

Proof. If the optimal contract features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then the second and third terms in the LHS of (21) are strictly negative and therefore optimism tightens the IC. This proves Lemma 13. ∎

Lemma 14. *If the agent believes signals are negatively correlated, i.e., ((2)) fails to hold, then the optimal contract implementing high effort features $c_{au} > c_{uu} = 0$.*

Proof. Recall that any optimal contract with truthful reporting for an agent who believes signals are negatively correlated satisfies either case (i) or case (ii) of Lemma 5. Assume case (i) of Lemma 5 holds then the IC becomes:

$$c_{au}\underbrace{(\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au})}_{>0}+c_{uu}\underbrace{(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}_{<0} \geq \Delta V.$$

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{au} > 0$ to always hold. Assume now case (ii) of Lemma 5 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. If $c_{au} > 0$, the Lemma is trivially proven. If $c_{aa} \geq 0$, case (ii) implies $c_{au} > c_{aa} \geq 0$. This proves the first part of Lemma 14.

To prove the second part of Lemma 14 we suppose it is false, i.e., at optimum the contract features $c_{uu} > 0$, and we prove that there exists a profitable deviation from it, which contradicts its optimality. From Lemma 5 we know that $c_{ua} \geq c_{uu}$ and also $c_{au} \geq c_{aa}$. The proof now depends on whether $c_{aa} > 0$ or $c_{aa} = 0$.

Let $c_{aa} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From the rearrangement of the constraint above we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{aa} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way the overall change in the LHS of the (IC) is given by:

$$\begin{aligned}
&-\epsilon\left(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au} + \Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}\right) \\
&\quad = -\epsilon\left(\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Delta\Gamma_a + \tilde{P}_{ua}\Delta\Gamma_u + \tilde{P}_{uu}\Delta\Gamma_u\right) \\
&\quad = -\epsilon\left(\Delta\Gamma_a + \Delta\Gamma_u\right) = -\epsilon\left(\Delta\Gamma_a - \Delta\Gamma_a\right) = 0
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, then the principal can decrease also w_{ua} and w_{aa} by the same amount. This does not violate the relevant (LL) and holds their difference constant. Hence, it does not violate any of the (TR_P) constraints. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{uu} > 0$ and $c_{aa} > 0$ cannot be the solution to the problem.

Let now, instead, the optimal contract feature $c_{aa} = 0$ and define $\Delta c_u = c_{ua} - c_{uu}$. We divide the proof for this case in three steps.

Step 1

When $c_{aa} = 0$, the (TR_A) imply:

$$\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq c_{au} \leq \Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}, \tag{27}$$

where, since we are in case (ii) of Lemma 5 either only one of the two inequalities holds as equality, or none. Suppose none of the two is strict, or the second one is, the principal can decrease both c_{ua} and c_{uu} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}) < 0$. He can then decrease c_{au} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{au}}$ bringing the LHS of the (IC) back to its original value. Clearly for some ϵ, this deviation can be done until the first inequality in (27) binds. Finally, to see that this is optimal for the principal, notice that according to the (LL) constraints, she can now decrease w_{uu} up to ϵ and w_{au} up to δ. By decreasing both by

$\min\{\epsilon,\delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected while the objective function decreases. This implies that at optimum if $c_{aa}=0$, the first inequality of (27) binds.

Step 2

Given that $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$ must hold at optimum if $c_{aa}=0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{aa}=0$ and $c_{uu}>0$. Let her decrease c_{ua} by ϵ and c_{uu} by $\epsilon_0<\epsilon$. Then Δc_u has decreased by $(\epsilon-\epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$, the principal decreases c_{au} by $(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$\begin{aligned}
& -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\Delta\tilde{\gamma}_{au} - \epsilon\Delta\tilde{\gamma}_{ua} - \epsilon_0\Delta\tilde{\gamma}_{uu} \\
= \; & -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{P}_{au}\Gamma^H_a}\tilde{P}_{au}\Delta\Gamma_a - \epsilon\tilde{P}_{ua}\Delta\Gamma_u - \epsilon_0\tilde{P}_{uu}\Delta\Gamma_u \\
= \; & -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\Delta\Gamma_a + \epsilon\tilde{P}_{ua}\Delta\Gamma_a + \epsilon_0\tilde{P}_{uu}\Delta\Gamma_a \\
= \; & \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{ua} - \frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\right) + \epsilon_0\left(\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a} + \tilde{P}_{uu}\right)\right] \\
= \; & \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{ua}\Gamma^H_a - \tilde{P}_{uu}\Gamma^H_u\right) + \epsilon_0\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{uu}\Gamma^H_a\right)\right] \\
= \; & \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{ua}\Gamma^H_a - \tilde{P}_{uu}(1-\Gamma^H_a)\right) + \epsilon_0\tilde{P}_{uu}\right] \\
= \; & \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon(\Gamma^H_a - \tilde{P}_{uu}) + \epsilon_0\tilde{P}_{uu}\right],
\end{aligned}$$

which is positive when:

$$\epsilon\left(\Gamma^H_a - \tilde{P}_{uu}\right) + \epsilon_0\tilde{P}_{uu} > 0.$$

If $\Gamma^H_a > \tilde{P}_{uu}$, the above is always true. If instead $\Gamma^H_a < \tilde{P}_{uu}$ then the principal has to choose $\epsilon \in \left\{\epsilon_0, \epsilon_0\frac{\tilde{P}_{uu}}{\tilde{P}_{uu}-\Gamma^H_a}\right\}$.

Step 3

To conclude, given the decreases in the c_{ts}, the principal can now decrease w_{uu} up to ϵ_0 and w_{au} up to $(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}$. By an argument similar to the one is Step 1, she can decrease both by the smallest of the two limits, decreasing the objective function. This provides the desired contradiction and hence, a contract where $c_{uu}>0$ and $c_{aa}=0$ cannot be the solution to the problem.

Finally, since a contract where $c_{uu}>0$ and $c_{aa}\geq 0$ cannot be a solution to the problem it follows that $c_{uu}=0$. This concludes the proof of the Lemma. ∎

Lemma 15. *If the agent believes signals are negatively correlated, i.e., ((2)) fails to hold, then constraint (TR^a_A) always binds in any optimal contract implementing high* *effort. Therefore:*

$$c_{ua} = \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}(c_{au} - c_{aa}).$$

Proof. Suppose not. Given the Lemmas and that the TR^a_A is slack, proven till now the problem that the principal faces is given by

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} \quad w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} + w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \tag{28}$$

$$\text{s.t.} \quad c_{aa}\Delta\tilde{\gamma}_{aa} + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} - \Delta V \geq 0 \tag{IC}$$

$$w_{aa}\gamma^H_{aa} + w_{au}\gamma^H_{au} \leq w_{ua}\gamma^H_{aa} + w_{uu}\gamma^H_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^H_{ua} + w_{uu}\gamma^H_{uu} \leq w_{aa}\gamma^H_{ua} + w_{au}\gamma^H_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^H_{aa} + c_{ua}\tilde{\gamma}^H_{ua} \geq c_{au}\tilde{\gamma}^H_{aa} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^H_{au} \geq c_{aa}\tilde{\gamma}^H_{au} + c_{ua}\tilde{\gamma}^H_{uu} \tag{TR^u_A}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

and we can rewrite the TR_P and TR_A constraints as

$$(w_{au} - w_{uu})\frac{\gamma^H_{au}}{\gamma^H_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^H_{uu}}{\gamma^H_{ua}} \tag{TR_P}$$

$$c_{ua}\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq (c_{au} - c_{aa}) \leq c_{ua}\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}. \tag{TR_A}$$

The principal can then decrease c_{ua} by ϵ, such that the TR^a_A still holds, and c_{au} and c_{aa} by $\epsilon\tilde{P}_{ua}$. Since the difference $c_{au} - c_{aa}$ is constant, the TR_A still hold. The IC is invariant since its LHS has changed by

$$-\epsilon\tilde{P}_{ua}\underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{\Delta\Gamma_a} - \epsilon\Delta\tilde{\gamma}_{ua} = \Delta\Gamma_a(\epsilon\tilde{P}_{ua} - \epsilon\tilde{P}_{ua}) = 0.$$

We are now left to show that this is optimal for the principal. Notice that both c_{ua} and c_{aa} have decreased. Hence, the principal can decrease both w_{ua} and w_{aa} by $\epsilon\tilde{P}_{ua}$. This does not violate TR_P and decreases the objective function, providing the desired contradiction. ∎

Given this, we can rewrite the IC as

$$c_{aa}\left(\Delta\tilde{\gamma}_{aa} - \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\Delta\tilde{\gamma}_{ua}\right) + c_{au}\left[\Delta\tilde{\gamma}_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\Delta\tilde{\gamma}_{ua}\right] - \Delta V > 0$$
$$c_{aa}\left(\Delta\Gamma_a\tilde{P}_{aa} + \frac{\tilde{\gamma}^H_{aa}}{\Gamma^H_u}\Delta\Gamma_a\right) + c_{au}\left(\Delta\Gamma_a\tilde{P}_{au} - \frac{\tilde{\gamma}^H_{aa}}{\Gamma^H_u}\Delta\Gamma_a\right) - \Delta V > 0$$
$$c_{aa}\left(\tilde{P}_{aa}\Gamma^H_u + \tilde{P}_{aa}\Gamma^H_a\right) + c_{au}\left(\tilde{P}_{au}\Gamma^H_u - \tilde{P}_{aa}\Gamma^H_a\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u$$
$$c_{aa}\tilde{P}_{aa}(\Gamma^H_a + \Gamma^H_u) + c_{au}\left[\tilde{P}_{au}(1-\Gamma^H_a) - \tilde{P}_{aa}\Gamma^H_a\right] > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u$$
$$c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \tilde{P}_{au}\Gamma^H_a - \tilde{P}_{aa}\Gamma^H_a\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u$$
$$c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma^H_a\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u$$

Lemma 16. *If the agent believes signals are negatively correlated, i.e., ((2)) fails to hold, then LL_{aa} and LL_{uu} bind in any optimal contract implementing high effort, i.e., $w_{aa} = c_{aa}$ and $w_{uu} = 0$.*

Proof. Consider the TR_P

$$\underbrace{(w_{au} - w_{uu})\frac{P_{au}}{P_{aa}}}_{LHS} \leq \underbrace{(w_{ua} - w_{aa})}_{\text{middle term}} \leq \underbrace{(w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}}}_{RHS}$$

Start from LL_{aa}. Suppose it does not bind. Then the principal can increase w_{au} by ϵ and decrease w_{aa} by $\epsilon_1 \equiv \epsilon\frac{P_{uu}}{P_{ua}}$. The values in TR_P change. The RHS increases by ϵ_1. The middle term also increases by ϵ_1. The LHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. To see that the LHS stays lower than the middle term notice that

$$\epsilon\frac{P_{au}}{P_{aa}} \leq \epsilon\frac{P_{uu}}{P_{ua}}$$

since

$$P_{au}P_{ua} < P_{aa}P_{uu}$$

by Assumption 2. Now notice that this creates an overall effect on the objective function given by

$$\epsilon\gamma^H_{au} - \epsilon\frac{P_{uu}}{P_{ua}}\gamma^H_{aa} = \epsilon\frac{1}{P_{ua}}\left(\gamma^H_{au}P_{ua} - \gamma^H_{aa}P_{uu}\right) = \epsilon\frac{1}{P_{ua}}(P_{au}P_{ua} - P_{aa}P_{uu})\Gamma^H_a < 0.$$

Hence this deviation contradicts the optimality of $w_{aa} > c_{aa}$.

For the LL_{uu} we follow the same logic. Suppose it does not bind. The principal can decrease w_{uu} by ϵ and increase w_{ua} by $\epsilon_1 \equiv \epsilon\frac{P_{au}}{P_{aa}}$. The values in TR_P change. The RHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. The middle term increases by ϵ_1. The LHS increases also by ϵ_1. To see that the RHS stays larger than the middle term notice that

$$\epsilon\frac{P_{au}}{P_{aa}} \leq \epsilon\frac{P_{uu}}{P_{ua}}$$

as above. Now notice that this creates an overall effect on the objective function given by

$$-\epsilon\gamma_{uu}^H + \epsilon\frac{P_{au}}{P_{aa}}\gamma_{ua}^H = \epsilon\frac{1}{P_{aa}}\left(\gamma_{ua}^H P_{au} - \gamma_{uu}^H P_{aa}\right) = \epsilon\frac{1}{P_{aa}}(P_{au}P_{ua} - P_{aa}P_{uu})\Gamma_u^H < 0.$$

Hence this deviation contradicts the optimality of $w_{uu} > c_{uu}$. ∎

Proof of Proposition 6

At optimum it is of course true that either TR_P^a or TR_P^u bind, or both. Since, however $P_{aa}P_{uu} - P_{au}P_{ua} > 0$ and $w_{uu} = 0$, the only way to have both binding would be for $w_{ua} = w_{aa}$ and $w_{au} = 0$. From Lemma 14, however, we know that $c_{au} > 0$. Hence, at least one for the two constraint has to be slack.

Constraint TR_P^a binding. Suppose the optimal contract sets the TR_P^a binding. We then have

$$w_{ua} = w_{au}\frac{P_{au}}{P_{aa}} + c_{aa}$$

which results in the following objective function

$$c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right).$$

Since w_{au} has a clear positive effect on it and the only constraint left on it is LL_{au}, we have that $w_{au} = c_{au}$. We can further simplify the objective function

$$\begin{aligned}
&c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right)\\
&= c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + c_{au}\frac{1}{P_{aa}}\left(P_{aa}P_{au}\Gamma_a^H + P_{au}P_{ua}\Gamma_u^H\right)\\
&= c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + c_{au}\frac{P_{au}}{P_{aa}}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)\\
&= c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + c_{au}\frac{P_{au}}{P_{aa}}\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\\
&= \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left[c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}\right]
\end{aligned}$$

which is equivalent to minimizing:

$$c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}.$$

This implies that iso-costs have slope $-P_{au}/P_{aa} < 0$.

On the other hand, the IC is not necessarily negatively sloped. Its slope is given by

$$-\frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}}$$

which is negative only if $b_a < P_{au} - \Gamma_a^H$.

The reduced problem for this case is given by

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} c_{aa} + c_{au}\frac{P_{au}}{P_{aa}} \tag{29}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa} \tag{TR_A^u}$$

Positively Sloped IC. Suppose $b_a > P_{au} - \Gamma_a^H$, the slope of the IC is positive and smaller than 1. To see this notice that

$$\Gamma_a^H - \tilde{P}_{au} < \tilde{P}_{aa} \Rightarrow \Gamma_a^H - 1 < 0$$

which is always true. Hence the binding constraints can be represented in (c_{au}, c_{aa}) space as in Figure 11.

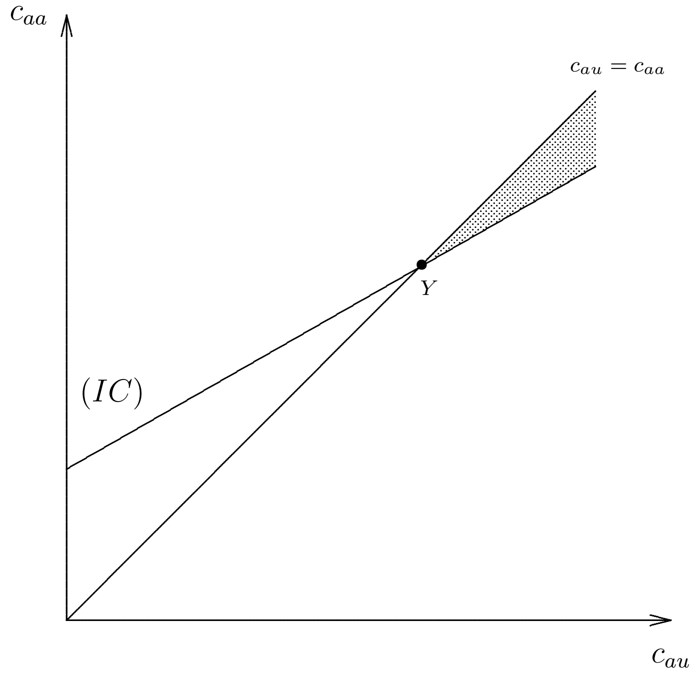

FIGURE 11. The shaded area represents the set of contracts satisfying all the constraints of the minimisation problem when the (IC) is positively sloped and the agent believes signals are negatively correlated.

In the Figure costs decrease towards the origin of the graph. The shaded area represents the set of contracts satisfying all constraints and the optimal contract is therefore at point Y. At Y, $c_{au} = c_{aa} > 0 = c_{ua}$. We derive the full contract below in Lemma 17 and show that it is equivalent to the BPE contract.

Negatively Sloped IC. Now suppose that $b_a < P_{au} - \Gamma_a^H$.[21] Then the problem can be represented as in Figure 12 below.

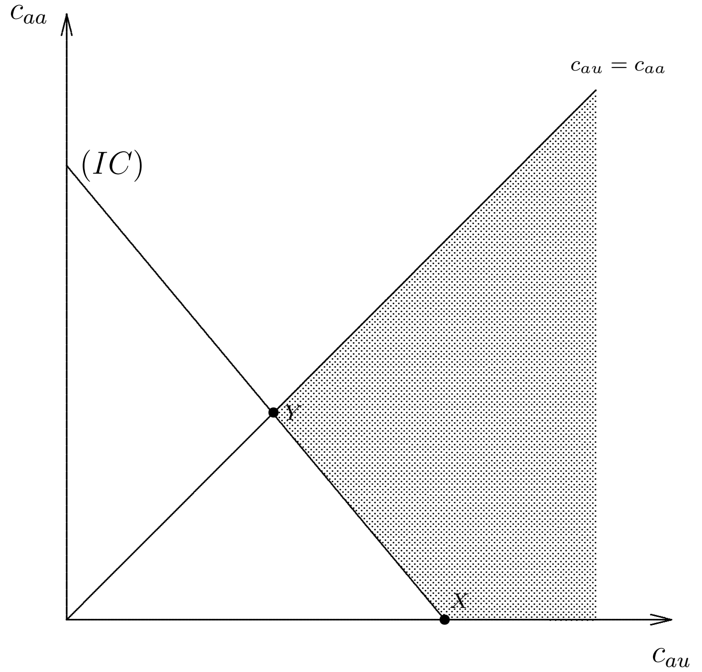

FIGURE 12. The shaded area represents the set of contracts satisfying all the constraints of the minimisation problem when the (IC) is negatively sloped and the agent believes signals are negatively correlated.

Once again, costs decrease towards the origin, but whether the minimum point lies at Y or X depends on the comparison between the slope of the IC and the one of the iso-costs, as in the case of an overconfident agent. In particular, the minimum lies at X if iso-costs are flatter than the IC. This happens when

$$\begin{aligned}
&\frac{P_{au}}{P_{aa}} \leq \frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}} \\
&P_{au}\tilde{P}_{aa} \leq \tilde{P}_{au}P_{aa} - \Gamma_a^H P_{aa} \\
&b_a P_{au} + b_a P_{aa} \leq P_{au}P_{aa} - \Gamma_a^H P_{aa} - P_{au}P_{aa} \\
&b_a(P_{au} + P_{aa}) \leq -\Gamma_a^H P_{aa} \\
&b_a \leq -P_{aa}\Gamma_a^H
\end{aligned} \tag{30}$$

Notice that $-P_{aa}\Gamma_a^H < P_{au} - \Gamma_A^H$. Hence (30) implies the negative slope of the IC.

Lemma 17. *If the agent believes signals are negatively correlated, i.e., ((2)) fails to hold, and (TR_P^a) binds, then the optimal contract implementing high effort is given by:*

$w_{aa} = c_{au} \quad w_{au} = c_{au} \quad w_{uu} = 0 \quad w_{ua} = \frac{c_{au}}{P_{aa}}$

$c_{aa} = c_{au} \quad c_{au} = \frac{\Delta V}{\Delta \Gamma_a} \quad c_{uu} = 0 \quad c_{ua} = 0.$

which fully replicates the BPE contract.

[21]Notice that this restriction may already fail for an optimistic agent if $P_{au} \leq \Gamma_a^H$.

Proof. Simply substitute $c_{aa} = c_{au}$ into the IC and notice that

$$c_{au}(\tilde{P}_{aa} + \tilde{P}_{au} - \Gamma_a^H) = \frac{\Delta V}{\Delta \Gamma_a} \Gamma_u$$

implies

$$c_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u}{(1 - \Gamma_a^H)} = \frac{\Delta V}{\Delta \Gamma_a}.$$

For w_{ua} notice that

$$w_{ua} = c_{au} \frac{P_{au}}{P_{aa}} + c_{aa} = c_{au} \left(\frac{P_{au}}{P_{aa}} + 1 \right) = c_{au} \left(\frac{P_{au} + P_{aa}}{P_{aa}} \right) = \frac{c_{au}}{P_{aa}}$$

∎

Since for an optimistic type b_a is never smaller or equal to $-P_{aa}\Gamma_a^H$, the BPE contract is the only possible contract for an optimistic agent who believes signals are negatively correlated, if the TR_P^a is binding.

No TR_P constraint binding (no deadweight loss contract). Suppose, now, all TR_P are slack. Then clearly all LL_{ts} constraint bind since the principal wants to decrease the expected wage paid as much as she can and they are the only constraints preventing her to set the $w_{ts} = 0$. We have

$$w_{ts} = c_{ts}, \quad c_{uu} = 0, \quad c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}(c_{au} - c_{aa}).$$

Then the principal solves

$$\min_{\{w_{ts}, c_{ts}\}_{t,s \in \{u,a\}}} \; c_{aa}\gamma_{aa}^H + c_{au}\gamma_{au}^H + \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}(c_{au} - c_{aa})\gamma_{ua}^H \tag{31}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa}. \tag{TR_A^u}$$

The sign of the slope of the iso-costs are not as trivial as above.

The objective function can be rearranged to obtain

$$\frac{1}{\tilde{\gamma}_{ua}^H} c_{aa}(\gamma_{aa}^H \tilde{\gamma}_{ua}^H - \tilde{\gamma}_{aa}^H \gamma_{ua}^H) + c_{au}(\gamma_{au}^H \tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H \gamma_{ua}^H)$$

which is equivalent to maximizing

$$c_{aa}(P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua}) + c_{au}(P_{au}\tilde{P}_{ua} + \tilde{P}_{aa}P_{ua}).$$

Hence the slope of the iso-costs is negative if:

$$\begin{aligned} &P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua} \\ &= b_u P_{aa} - b_a P_{ua} > 0. \end{aligned} \tag{32}$$

Now notice two things

(1) If the IC is positively sloped the optimal point would be at $c_{au} = c_{aa}$, regardless of whether costs decrease towards the origin (negatively sloped iso-costs) or towards the top-left corner (positively sloped) in (c_{au}, c_{aa}) space. This, however, yields an unfeasible contract since $c_{au} = c_{aa} \Rightarrow c_{ua} = 0 = w_{ua}$, which violates the TR_P^a constraint since

$$c_{au}\frac{P_{au}}{P_{aa}} > -c_{au}.$$

(2) If the IC is negatively sloped, the constraint of the problem are the same as the ones represented already in Figure 12. When the iso-costs are positively sloped, or when they are negatively sloped but steeper than the IC, the optimal point would be at $c_{au} = c_{aa} (\Rightarrow c_{ua} = 0 = w_{ua})$ again.

These two observations imply that the only possible feasible contract for this case is one where the iso-costs and the IC are negatively sloped and the former are flatter than the latter.[22] This happens when

$$\begin{aligned}
&\frac{P_{au}\tilde{P}_{ua} + \tilde{P}_{aa}P_{ua}}{P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua}} \leq \frac{P_{au} - b_a - \Gamma_a^H}{P_{aa} + b_a}, \\
&\frac{(P_{au} + P_{aa})P_{ua} + b_u P_{au} + b_a P_{ua}}{P_{aa}b_u - b_a P_{ua}} \leq \frac{P_{au} - b_a - \Gamma_a^H}{P_{aa} + b_a}, \\
&(P_{ua} + b_u P_{au} + b_a P_{ua})(P_{aa} + b_a) \leq (P_{aa}b_u - b_a P_{ua})(P_{au} - b_a - \Gamma_a^H), \\
&P_{aa}P_{ua} + b_u P_{aa}P_{au} + b_a P_{ua}P_{aa} + b_a P_{ua} + b_a b_u P_{au} + b_a^2 P_{ua} \\
&\qquad\qquad\qquad \leq \\
&b_u P_{aa}P_{au} - b_a b_u P_{aa} - b_u \Gamma_a^H P_{aa} - b_a P_{ua}P_{au} + b_a^2 P_{ua} + b_a \Gamma_a^H P_{ua}, \\
&P_{aa}P_{ua} + b_a P_{ua} + b_a P_{ua}\Gamma_u^H + b_a b_u + b_u \Gamma_a^H P_{aa} \geq 0 \\
&b_a(P_{ua}(1 + \Gamma_u^H) + b_u) \leq -P_{aa}\left(P_{ua} + b_u \Gamma_a^H\right)
\end{aligned}$$

which generates

$$b_a \leq -P_{aa}\frac{\left(P_{ua} + b_u \Gamma_a^H\right)}{(P_{ua}(1 + \Gamma_u^H) + b_u)} \tag{33}$$

that always fails for $b_a < 0$.[23]

Constraint TR_P^u binding. Suppose now we are in the case of TR_P^u binding. We have

$$w_{ua} = w_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}.$$

[22] Notice that since we assumed that the TR_P are slack, they cannot be considered as restrictions to the problem. On the contrary when we assumed the TR_P^a binding in the previous case we made no assumption about the LL_{ts} and therefore they were considered as potentially binding.

[23] Notice that $(P_{ua}(1 + \Gamma_u^H) + b_u) > 0$ is always true since

$$-P_{ua}(1 + \Gamma_u^H) < -P_{ua} < b_u.$$

In this case the objective function is given by

$$
\begin{aligned}
& c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+w_{au}\left(\gamma^H_{au}+\frac{P_{uu}}{P_{ua}}P_{ua}\Gamma^H_u\right)\\
&=c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+w_{au}\left(\gamma^H_{au}+P_{uu}\Gamma^H_u\right)\\
&=c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+w_{au}\left(\gamma^H_{au}+\gamma^H_{uu}\right)
\end{aligned}
$$

Since w_{au} has a clear positive effect on it and the only constraint left on it is LL_{au}, we have that $w_{au}=c_{au}$. The reduced problem for this case is therefore given by

$$
\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+c_{au}\left(\gamma^H_{au}+\gamma^H_{uu}\right) \tag{34}
$$

$$
\text{s.t.} \quad c_{aa}\tilde{P}_{aa}+c_{au}\left(\tilde{P}_{au}-\Gamma^H_a\right)>\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u \tag{IC}
$$

$$
c_{au}\geq c_{aa}. \tag{TR^u_A}
$$

We can immediately see that iso-costs are always negatively sloped.

Lemma 18. *If the agent is optimistic and believes signals are negatively correlated, then there exists no optimal contract implementing high effort where* (TR^u_P) *binds.*

Proof. Suppose not, and the TR^u_P binds. Suppose $b_a>P_{au}-\Gamma^H_a$, the IC are positively sloped and Figure 11 represents again the constraints of the problem. The optimal contract would feature $c_{ua}=0$ and the contract resemble the one of Lemma 17 with the only difference that

$$
w_{ua}=c_{au}\frac{P_{uu}}{P_{ua}}+c_{au}=\frac{c_{au}}{P_{ua}}.
$$

However, since $P_{ua}<P_{aa}$ (from Assumption 2) this contract is clearly dominated by the BPE contract in Lemma 17.

If instead the IC is negatively sloped, we are, once again, in Figure 12, where a new contract may arise if iso-costs are flatter than the IC (if instead they are steeper we have the BPE contract again, for the reasons just explained). As standard by now, we are going to show that this case can never happen if the agent is optimistic. Iso-costs are flatter than the IC if

$$
(P_{ua}+b_u)(\gamma^H_{au}+\gamma^H_{uu})<(\tilde{P}_{au}-\Gamma^H_a)(\gamma^H_{ua}+\gamma^H_{aa}).
$$

Notice that the condition becomes looser the smaller is b_u. Since the agent believes signals to be negatively correlated, from (2), b_u must be at least $P_{aa}+b_a-P_{ua}=$

$\tilde{P}_{aa} - P_{ua}$. Hence we check the above assuming the floor value of b_u.

$$\begin{aligned}
&(P_{ua} + \tilde{P}_{aa} - P_{ua})(\gamma^H_{au} + \gamma^H_{uu}) < (\tilde{P}_{au} - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa}) \\
&\tilde{P}_{aa}(\gamma^H_{au} + \gamma^H_{uu}) < (1 - \tilde{P}_{aa} - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa}) \\
&\tilde{P}_{aa}(\gamma^H_{au} + \gamma^H_{uu} + \gamma^H_{ua} + \gamma^H_{aa}) < (1 - \Gamma^H_a)(\gamma^H_{ua} + \gamma^H_{aa}) \\
&\tilde{P}_{aa} < \Gamma^H_u(\gamma^H_{ua} + \gamma^H_{aa}) \\
&P_{aa} + b_a - \Gamma^H_u(P_{ua}\Gamma^H_u + P_{aa}\Gamma^H_a) < 0 \\
&P_{aa}(1 - \Gamma^H_a\Gamma^H_u) + b_a - P_{ua}\left(\Gamma^H_u\right)^2 < 0.
\end{aligned}$$

where we know that $P_{aa} > P_{ua}$ by Assumption 2 and we can calculate

$$(1 - \Gamma^H_a\Gamma^H_u) = 1 - \Gamma^H_A + \left(\Gamma^H_a\right)^2 > 1 - \Gamma^H_a > \left(1 - \Gamma^H_a\right)^2 = \left(\Gamma^H_u\right)^2.$$

Hence the LHS is always positive for $b_a > 0$ and the condition can never been satisfied for an optimistic agent who believes signals are negatively correlated. This concludes the proof. ∎

Proof of Proposition 7

To see this consider the proof of Proposition 2 and notice that everything follows through in this case as well until condition (22) which can never hold for $b_a < 0$.

Proofs of Propositions 8 and 9

To prove the Propositions we build on the findings of the proof of Proposition 6. First, we show that when (11) and (12) hold, the PED-DL is the optimal contract set up. Second we show that when (11) fails but (13) and (14) hold the PED-NDL is feasible and optimal. Finally, we show how Lemma 18 holds in this case too. The proofs described here are true also for the results for a skeptical agent highlighted in section 6.

Start from the case where TR^a_P binds and notice that (30) (which is the equivalent of (12) and will be denoted 12 from here on) may now hold since the agent's bias features $b_a < 0$.

From Figure 12 we see that when (12) holds, $c_{aa} = 0$, c_{ua} follows from Lemma 15 and to find c_{au} we calculate

$$c_{au}(\tilde{P}_{au} - \Gamma^H_a) = \frac{\Delta V}{\Delta \Gamma_a}\Gamma_u \Rightarrow c_{au} = \frac{\Delta V}{\Delta \Gamma_a}\frac{\Gamma_u}{\tilde{P}_{au} - \Gamma^H_a}.$$

Notice however, that given the value of w_{ua}, whether LL_{ua} holds or not is not straightforward. Hence, we have

$$
\begin{aligned}
& w_{ua} \geq c_{ua} \\
& \frac{P_{au}}{P_{aa}} c_{au} \geq \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au} \\
& \frac{P_{au}}{P_{aa}} \geq \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} \\
& P_{au} \tilde{\gamma}^H_{ua} \geq \tilde{\gamma}^H_{aa} P_{aa} \\
& P_{au} \tilde{P}_{ua} \Gamma^H_u \geq P_{aa} \tilde{P}_{aa} \Gamma^H_a \\
& P_{au} \tilde{P}_{ua} \Gamma^H_u - P_{aa} \tilde{P}_{aa} \Gamma^H_a \geq 0 \\
& P_{au} P_{ua} \Gamma^H_u - P^2_{aa} \Gamma^H_a + b_u P_{au} \Gamma^H_u - b_a P_{aa} \Gamma^H_a \geq 0
\end{aligned}
$$

which generates (11).

Now suppose no TR_P constraint holds and notice that (33) (which is equivalent to (14)) may now hold. When it does we have the same contract of PED-DL with the difference that now w_{ua} derives from the LL_{ua} instead of the TR^a_P and therefore

$$
w_{ua} = c_{ua} = \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au}.
$$

Further checks have to be carried out to be sure that the contract satisfies the TR_P constraints. We start from the TR^a_P and see that it holds as long as

$$
\begin{aligned}
& w_{ua} > \frac{P_{au}}{P_{aa}} w_{au} \Rightarrow c_{ua} > \frac{P_{au}}{P_{aa}} c_{au} \\
& \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au} > \frac{P_{au}}{P_{aa}} c_{au} \Rightarrow \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} \frac{P_{aa}}{P_{au}} c_{au} > c_{au} \\
& \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} \frac{P_{aa}}{P_{au}} \geq 1 \Rightarrow P_{aa} \tilde{\gamma}^H_{aa} - P_{au} \tilde{\gamma}^H_{ua} \geq 0 \\
& P_{aa} \tilde{P}_{aa} \Gamma^H_a - P_{au} \tilde{P}_{ua} \Gamma^H_u > 0
\end{aligned}
$$

which yields the opposite of (11).

Now we check for TR_P^u to hold

$$\begin{aligned}
& w_{ua} \leq \frac{P_{uu}}{P_{ua}} w_{au} \;\Rightarrow\; c_{ua} < \frac{P_{uu}}{P_{ua}} c_{au} \\
& \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au} < \frac{P_{uu}}{P_{ua}} c_{au} \;\Rightarrow\; \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} \frac{P_{uu}}{P_{ua}} c_{au} < c_{au} \\
& \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} \frac{P_{uu}}{P_{ua}} \leq 1 \;\Rightarrow\; P_{uu}\tilde{\gamma}^H_{ua} - P_{ua}\tilde{\gamma}^H_{aa} > 0 \\
& P_{uu}\tilde{P}_{ua}\Gamma^H_u - P_{ua}\tilde{P}_{aa}\Gamma^H_a > 0 \\
& P_{uu}P_{ua}\Gamma^H_u - P_{ua}P_{aa}\Gamma^H_a + b_u P_{uu}\Gamma^H_u - b_a P_{ua}\Gamma^H_a > 0 \\
& b_u P_{uu}\Gamma^H_u - b_a P_{ua}\Gamma^H_a > P_{aa}P_{ua}\Gamma^H_a - P_{uu}P_{ua}\Gamma^H_u
\end{aligned}$$

which yields (13).

Before proving that no optimal contract exists where TR_P^u binds, notice that the above contracts are feasible in completely distinct areas (since (11) separates them) and that if all the conditions derived hold, they also "dominate" the BPE, they are therefore optimal.

To conclude the proof we provide a different proof to Lemma 18. From the original proof, notice that it is possible now for the iso-costs to be flatter than the IC. However, we now show that (i) the resulting contract with $c_{aa} = 0$ and TR_P^u binding is feasible only if (13) holds, (ii) it is always dominated by the contract without a deadweight loss derived above when the latter is feasible, (iii) it is always dominated by the contract with TR_P^a binding derived above when the latter is feasible. Hence, this new contract, even if optimal given the assumption of TR_P^u binding, is never generally optimal and can be ignored.

Let's start from (i). Notice that the contract lying at point X of Figure 12 for this case is given by

$$\begin{array}{llll}
w_{aa} = 0 & w_{au} = c_{au} & w_{uu} = 0 & w_{ua} = \frac{P_{uu}}{P_{\mu a}} c_{au} \\
c_{aa} = 0 & c_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a} & c_{uu} = 0 & c_{ua} = \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au}
\end{array}$$

The calculations follow the same identical derivations of the case of TR_P^a binding but for w_{ua} which simply follows from $w_{ua} = c_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}$ given by the TR_P^u.

Given this, constraint LL_{ua} holds if

$$\frac{P_{uu}}{P_{ua}} c_{au} \geq \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}} c_{au}$$

which generates (13) again.

For (ii) we compare the average wage payment in both contracts. Without the need of any algebra, we notice that the no deadweight loss contract features $w_{ts} = c_{ts}$ for all t and s and the compensations and wages offered by the two contracts are identical but for w_{ua}. Hence, the only way for the contract with TR_P^u binding to grant a lower expected wage payment than the no deadweight loss contract is for it to feature $w_{ts} < c_{ts}$ for some ts, which is infeasible.

Finally, for (iii) notice that the two contracts again feature identical c_{ts} and w_{ts} but for w_{ua}. The contract with TR_P^a binding grant a lower average wage payment if

$$\frac{P_{au}}{P_{aa}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right) \leq \frac{P_{uu}}{P_{ua}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right)$$

which boils down to simply

$$P_{au}P_{ua} - P_{uu}P_{aa} \leq 0$$

which is always true.

This proves both propositions and holds for a skeptical agent as well.

Proof of Corollary 1

First of all notice that the only difference between a PED-DL and an PED-NDL contract lies in the wages. Hence

$$\sum_{ts} \hat{c}_{ts}\tilde{\gamma}_{ts} = \sum_{ts} \hat{c}'_{ts}\tilde{\gamma}_{ts}.$$

Therefore, to prove the Lemma, we need to check that

$$\min\left\{\sum_{ts} c^*_{ts}\tilde{\gamma}_{ts} - V(\lambda^H), \sum_{ts} c^\dagger_{ts}\tilde{\gamma}_{ts} - V(\lambda^H), \sum_{ts} \hat{c}_{ts}\tilde{\gamma}_{ts} - V(\lambda^H)\right\} \geq \bar{u}$$

Our welfare analysis in section 7 shows that

$$\min\left\{\tilde{E}(c^*_{ts}), \tilde{E}(c^\dagger_{ts}), \tilde{E}(\hat{c}_{ts})\right\} = \tilde{E}(c^*_{ts}).$$

Hence, it is enough to show that the BPE satisfies the PC. From the BPE contracts we can derive:

$$\begin{aligned}
\tilde{E}(c^*_{ts}) &= c_{au}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) \\
&= \frac{\Delta V}{\Delta\Gamma_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) \\
&= \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a
\end{aligned}$$

hence we simply check that

$$\begin{aligned}
\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a - V(\lambda^H) &\geq \bar{u} \\
\left(\frac{\Gamma^H_a}{\Delta\Gamma_a} - 1\right)V(\lambda^H) - V(\lambda^L)\frac{\Gamma^H_a}{\Delta\Gamma_a} &\geq \bar{u} \\
\frac{\Gamma^L_a}{\Delta\Gamma_a}V(\lambda^H) - V(\lambda^L)\frac{\Gamma^H_a}{\Delta\Gamma_a} &\geq \bar{u}
\end{aligned}$$

which yields

$$\bar{u} \leq \frac{V(\lambda^H)\Gamma_a^L - V(\lambda^L)\Gamma_a^H}{\Delta\Gamma_a}$$

and proves the Corollary.

Proof of Proposition 10

While $c_{au}^\dagger < c_{aa}^*$ we also have that $c_{aa}^\dagger > c_{au}^\dagger$. Therefore the check for $c_{aa}^\dagger > c_{aa}^*$ is given by:

$$\left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)\left(\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)}\right) \geq 1$$

which is equivalent to

$$\left(\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H}\right)\left(\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)}\right) \geq 1$$

and to

$$\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(1 - \Gamma_A^H)}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} \geq 1$$

Which is always true since $\tilde{P}_{aa} \leq 1$.

To prove that $\max\{\hat{c}_{au}, \hat{c}_{ua}\} > c_{au}^*$ we simply check that

$$\frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} > 1$$

which yields

$$\underbrace{\Gamma_a^H + \Gamma_u^H}_{1} - \tilde{P}_{au}$$

which is always true.

Proof of Proposition 11

Point (i) is trivial. Condition (9) comes from the study of how to minimize cost and it selects the optimal contract precisely on the basis of the lowest possible expected wage. Since both contracts are available at the moment of minimization none of the two can minimize costs when the other is optimal.

To prove point (ii) notice that

$$\begin{aligned} E(c_{ts}^*) &= c_{aa}^*\gamma_{aa}^H + c_{au}^*\gamma_{au}^H + c_{ua}^*\gamma_{ua}^H + c_{uu}^*\gamma_{uu}^H \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\gamma_{aa}^H + \gamma_{au}^H) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H, \end{aligned}$$

and

$$\begin{aligned} \tilde{E}(c_{ts}^*) &= c_{aa}^*\tilde{\gamma}_{aa}^H + c_{au}^*\tilde{\gamma}_{au}^H + c_{ua}^*\tilde{\gamma}_{ua}^H + c_{uu}^*\tilde{\gamma}_{uu}^H \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\tilde{\gamma}_{aa}^H + \tilde{\gamma}_{au}^H) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H, \end{aligned}$$

where we used the fact that $\gamma^H_{ta} + \gamma^H_{tu} = \tilde{\gamma}^H_{ta} + \tilde{\gamma}^H_{tu} = \Gamma^H_t$ (which is easily proven from Lemma 1 and Assumption 3).

Point (iii) requires us to calculate $\tilde{E}(c^\dagger_{ts})$.

$$\begin{aligned}
\tilde{E}(c^\dagger_{ts}) &= c^\dagger_{aa}\tilde{\gamma}^H_{aa} + c^\dagger_{au}\tilde{\gamma}^H_{au} + c^\dagger_{ua}\tilde{\gamma}^H_{ua} + c^\dagger_{uu}\tilde{\gamma}^H_{uu} \\
&= c^\dagger_{au}\left[\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right] \\
&= \frac{c^\dagger_{au}}{\tilde{\gamma}^H_{au}}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au})(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) \\
&= \frac{c^\dagger_{au}}{\tilde{\gamma}^H_{au}}\Gamma^H_a(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) \\
&= \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a \frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} \\
&= \tilde{E}(c^*_{ts})\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}.
\end{aligned}$$

Since $\Gamma^H_u = 1 - \Gamma^H_a$, it is clear that the numerator is at least as large as the denominator. This proves point (iii).

Finally, for point (iv) we need to calculate $E(c^\dagger_{ts})$.

$$\begin{aligned}
E(c^\dagger_{ts}) &= c^\dagger_{aa}\gamma^H_{aa} + c^\dagger_{au}\gamma^H_{au} + c^\dagger_{ua}\gamma^H_{ua} + c^\dagger_{uu}\gamma^H_{uu} \\
&= c^\dagger_{au}\left[\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\gamma^H_{aa} + \gamma^H_{au} + \gamma^H_{uu}\right] \\
&= \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma^H_{aa}\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\gamma^H_{aa} + \gamma^H_{au}\tilde{\gamma}^H_{au} + \gamma^H_{uu}\tilde{\gamma}^H_{au}}{\tilde{P}_{au}\Gamma^H_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} \\
&= \frac{\Delta V}{\Delta \Gamma_a}\frac{\tilde{\gamma}_{au}\Gamma^H_a + \tilde{\gamma}_{uu}P_{aa}\Gamma^H_a + \gamma^H_{uu}\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} \\
&= E(c^*_{ts})\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}.
\end{aligned}$$

Hence, to prove our result we are left to show that

$$\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} > 1$$

which is equivalent to

$$\tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u \geq \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a).$$

This requires some calculations.

$$\tilde{P}_{uu}P_{aa}(1-\Gamma_a^H)+P_{uu}(1-\tilde{P}_{aa})(1-\Gamma_a^H)-\tilde{P}_{uu}\tilde{P}_{aa}+\tilde{P}_{uu}\Gamma_a^H \geq 0$$
$$\tilde{P}_{uu}P_{aa}-\tilde{P}_{uu}P_{aa}\Gamma_a^H+P_{uu}-P_{uu}\tilde{P}_{aa}-P_{uu}\Gamma_a^H+P_{uu}\tilde{P}_{aa}\Gamma_a^H-\tilde{P}_{uu}\tilde{P}_{aa}+\tilde{P}_{uu}\Gamma_a^H \geq 0$$
$$\tilde{P}_{uu}P_{aa}-P_{uu}\tilde{P}_{aa}-\tilde{P}_{uu}\tilde{P}_{aa}-P_{uu}\Gamma_a^H+\tilde{P}_{uu}\Gamma_a^H-\tilde{P}_{uu}P_{aa}\Gamma_a^H+P_{uu}\tilde{P}_{aa}\Gamma_a^H+P_{uu} \geq 0$$

From here, we substitute for some of the $\tilde{P}_{ts}$ to get

$$\begin{aligned}(\tilde{P}_{uu}P_{aa}-P_{uu}\tilde{P}_{aa}-\tilde{P}_{uu}P_{aa}-\tilde{P}_{uu}b_a)+(-P_{uu}\Gamma_a^H+P_{uu}\Gamma_a^H-b_u\Gamma_a^H)+\\+(-P_{uu}P_{aa}\Gamma_a^H+b_uP_{aa}\Gamma_a^H+P_{uu}P_{aa}\Gamma_a^H+P_{uu}b_a\Gamma_a^H)+P_{uu} \geq 0\end{aligned}$$

and finally

$$-P_{uu}\tilde{P}_{aa}-\tilde{P}_{uu}b_a-b_u\Gamma_a^H+P_{uu}b_a\Gamma_a^H+b_uP_{aa}\Gamma_a^H+P_{uu} \geq 0$$
$$-P_{uu}P_{aa}-P_{uu}b_a-P_{uu}b_a+b_ub_a-b_u\Gamma_a^H+P_{uu}b_a\Gamma_a^H+b_uP_{aa}\Gamma_a^H+P_{uu} \geq 0$$
$$-P_{uu}\underbrace{(P_{aa}+b_a)}_{1-\tilde{P}_{au}}-P_{uu}b_a+b_ub_a-b_u\Gamma_a^H+P_{uu}b_a\Gamma_a^H+b_uP_{aa}\Gamma_a^H+P_{uu} \geq 0$$
$$b_u\left(b_a-\Gamma_a+P_{aa}\Gamma_a^H\right)+P_{uu}\left[1-b_a(1-\Gamma_a^H)-(1-\tilde{P}_{au})\right] \geq 0$$
$$b_u\left(b_a-P_{au}\Gamma_a^H\right) \geq P_{uu}\left[b_a(1-\Gamma_a^H)+1-\tilde{P}_{au}-1\right]$$
$$b_u\left(b_a-P_{au}\Gamma_a^H\right) \geq P_{uu}\left[b_a(1-\Gamma_a^H)-P_{au}+b_a\right]$$
$$b_u\left(b_a-P_{au}\Gamma_a^H\right) \geq P_{uu}\left[b_a\Gamma_u^H-P_{au}+b_a\right]$$
$$b_u\left(b_a-P_{au}\Gamma_a^H\right) \geq P_{uu}\left(b_a(1+\Gamma_u^H)-P_{au}\right)$$

Notice now that the APE requires $b_a > P_{au}\Gamma_a$ as described in the proof of Proposition 5. This means that the LHS is always positive and we can therefore derive the condition presented in the Result.

Proof of Proposition 12

First we study condition (15).

$$b_u \geq P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}$$

At $b_u = 0$, condition (15) corresponds to $b_a < P_{au}/(1+\Gamma_u^H)$. Hence, $P_{au}/(1+\Gamma_u^H)$ is the intercept of the RHS of the condition with the x-axis. Let

$$P_{au}/(1+\Gamma_u) \equiv \underline{b}_a.$$

To show that this condition is compatible with (9), and therefore that an area where optimism is socially desirable always exists, we need to show that $\underline{b}_a$ is larger than the intercept of condition (9) (holding with equality) with the x-axis. We start from the latter, which we already calculated in Part 3 of the proof to Proposition 5.

$$b_a = P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}.$$

We then need to show that

$$P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z} < \frac{P_{au}}{(1+\Gamma_u^H)}.$$

To do this, we get

$$\begin{aligned}
&(1-\Gamma_a^H)W + P_{uu}Z > (1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z + (1-\Gamma_a^H)W\Gamma_u^H + P_{uu}\Gamma_a^H Z\Gamma_u^H \\
&P_{uu}(1-\Gamma_a^H)Z - (1-\Gamma_a^H)W\Gamma_u^H - P_{uu}\Gamma_a^H\Gamma_u^H Z > 0 \\
&P_{uu}\Gamma_u^H Z - W(\Gamma_u^H)^2 - P_{uu}\Gamma_a^H\Gamma_u^H Z > 0 \\
&P_{uu}\Gamma_u^H Z \underbrace{(1-\Gamma_a^H)}_{\Gamma_u^H} - W(\Gamma_u^H)^2 > 0 \;\Rightarrow\; P_{uu}Z - W > 0
\end{aligned}$$

We can now expand Z and W to get

$$\begin{aligned}
&P_{uu}Z - W > 0 \\
&P_{uu}P_{aa}\Gamma_a^H + P_{uu}P_{ua}\Gamma_u^H - \Gamma_a^H P_{aa} + \Gamma_a^H P_{ua} > 0 \\
&\underbrace{(P_{uu}-1)}_{-P_{ua}} P_{aa}\Gamma_a^H + P_{uu}P_{ua}\Gamma_u^H + \Gamma_a^H P_{ua} > 0 \\
&-P_{aa}\Gamma_a^H + P_{uu}\Gamma_u^H + \Gamma_a^H > 0 \\
&P_{uu}\Gamma_u^H + \Gamma_a^H(1-P_{aa}) > 0
\end{aligned}$$

which is obviously always true. This proves that an area where optimism is socially desirable always exists, at least for $b_u = 0$. We now show that this area also exists for positive values of b_u. To do this, consider the shape of condition (9) as in Figure 5. Since we know that the curve of condition (9) intercepts the x-axis before (15), it is enough to show that the loci of points where the two conditions hold cross only once in (b_a, b_u) space and they do so at $(b_a, b_u) = (P_{au}, P_{uu})$. To formally prove the shape of Figure 7 we are also going to show that the locus where (15) binds is concave in (b_a, b_u) space.

Take the two conditions binding and equate the two RHSs to get:

$$
\begin{aligned}
& P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}=P_{uu}-\frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a-P_{au}\Gamma_a^H\right)Z}\\
& P_{uu}\left[(b_a(1+\Gamma_u^H)-P_{au})Z-(b_a-P_{au}\Gamma_a^H)Z\right]=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
& P_{uu}\left[b_a\Gamma_u^H Z-P_{au}\Gamma_u^H Z\right]=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
& P_{uu}(b_a-P_{au})\Gamma_u^H Z=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
& (P_{uu}Z-W)(b_a-P_{au})=0\\
& (P_{uu}P_{aa}\Gamma_a^H+P_{uu}P_{ua}\Gamma_u^H-P_{aa}\Gamma_a^H+P_{ua}\Gamma_a^H)(b_a-P_{au})=0\\
& ((P_{uu}-1)P_{aa}\Gamma_a^H+P_{ua}(P_{uu}\Gamma_u^H+\Gamma_a^H))(b_a-P_{au})=0\\
& P_{ua}(-P_{aa}\Gamma_a^H+P_{uu}\Gamma_u^H+\Gamma_a^H)(b_a-P_{au})=0\\
& P_{ua}(P_{au}\Gamma_a^H+P_{uu}\Gamma_u^H)(b_a-P_{au})=0
\end{aligned}
$$

which holds only if $b_a=P_{au}$.When plugged into any of the two conditions we get that the corresponding value is $b_u=P_{uu}$. Hence the two curves cross only at that point. This concludes the proof of the Proposition. To show that the RHS of (15) is concave simply calculate the first derivative and obtain:

$$
\begin{aligned}
&\frac{\partial}{\partial b_a}\left[P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}\right]\\
&=P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}\Gamma_a^H(1+\Gamma_u^H)-b_a(1+\Gamma_u^H)+P_{au}}{(b_a-P_{au}\Gamma_a^H)^2}\\
&=P_{uu}P_{au}\frac{1-\Gamma_a^H(1+\Gamma_u^H)}{(b_a-P_{au}\Gamma_a^H)^2}\\
&=P_{uu}P_{au}\frac{1-2\Gamma_a^H+(\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}=P_{uu}P_{au}\frac{(1-\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}>0.
\end{aligned}
$$

The second derivative is obviously negative since b_a only appears at the denominator.

Proof of Proposition 13

Point (i) follows from the fact that the PED contracts feature the same wage but for the $ts=ua$ case and the optimality of the PED contracts (as in Proposition 11).

Point (ii)'s equality is straightforward. To see why the inequality is true we calculate

$$\begin{aligned}
\tilde{E}(\hat{c}_{ts}) =& \hat{c}_{aa}\tilde{\gamma}^H_{aa} + \hat{c}_{au}\tilde{\gamma}^H_{au} + \hat{c}_{ua}\tilde{\gamma}^H_{ua} + \hat{c}_{uu}\tilde{\gamma}^H_{uu} \\
=& \hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\tilde{\gamma}^H_{ua}\right) \\
=& \hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{aa}\right) = \hat{c}_{au}\Gamma^H_a \\
=& \frac{\Delta V}{\Delta\Gamma^H_a}\frac{\Gamma^H_a\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a}.
\end{aligned}$$

Hence to prove point (ii) we simply need

$$\frac{\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a} > 1 \quad \Rightarrow \quad \underbrace{\Gamma^H_a + \Gamma^H_u}_{1} - \tilde{P}_{au} > 0$$

which always holds.

Finally, to prove point (iii) we calculate

$$\begin{aligned}
E(\hat{c}_{ts}) =& \hat{c}_{aa}\gamma^H_{aa} + \hat{c}_{au}\gamma^H_{au} + \hat{c}_{ua}\gamma^H_{ua} + \hat{c}_{uu}\gamma^H_{uu} \\
=& \hat{c}_{au}\left(\gamma^H_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\gamma^H_{ua}\right) \\
=& \frac{\Delta V}{\Delta\Gamma^H_a}\frac{\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua}}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}}.
\end{aligned}$$

Hence to prove point (iii) we need

$$\begin{gathered}
\frac{\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua}}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}} > \Gamma^H_a \\
\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua} > \Gamma^H_a(\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}) \\
P_{au}\tilde{P}_{ua}\Gamma^H_a\Gamma^H_u + \tilde{P}_{aa}P_{ua}\Gamma^H_a\Gamma^H_u > \Gamma^H_a(\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}) \\
P_{au}\tilde{P}_{ua}\Gamma^H_u + \tilde{P}_{aa}P_{ua}\Gamma^H_u - \tilde{P}_{au}\tilde{P}_{ua} + \Gamma^H_a\tilde{P}_{ua} > 0 \\
\underbrace{P_{au}P_{ua}\Gamma^H_u + P_{aa}P_{ua}\Gamma^H_u}_{P_{ua}\Gamma^H_u} - P_{au}P_{ua} + \Gamma^H_a P_{ua} \\
+ b_u P_{au}\Gamma^H_u + b_a P_{ua}\Gamma^H_u + b_a P_{ua} - b_u P_{au} + b_a b_u + b_u\Gamma^H_a > 0 \\
P_{ua}(\Gamma^H_u - P_{au} + \Gamma^H_a) + b_u(P_{au}\Gamma^H_u - P_{au} + \Gamma^H_a) + b_a(P_{ua}\Gamma^H_u + P_{ua} + b_u) > 0 \\
P_{ua}(1 - P_{au}) + b_u\left(P_{au}\underbrace{(\Gamma^H_u - 1)}_{\Gamma^H_a} + \Gamma^H_a\right) + b_a\left(P_{ua}(1 + \Gamma^H_u) + b_u\right) > 0 \\
P_{ua}P_{aa} + b_u\Gamma^H_a(1 - P_{au}) + b_a\left(P_{ua}(1 + \Gamma^H_u) + b_u\right) > 0 \\
b_a\left(P_{ua}(1 + \Gamma^H_u) + b_u\right) > -P_{ua}P_{aa} - b_u\Gamma^H_a P_{aa}
\end{gathered}$$

which generates the opposite of (14).

Proof of Proposition 14

The first statement is trivial since the PED-NDL contract is optimal only if (14) and it would be socially desirable only when (14) fails. Hence, the PED-NDL contract never Pareto improves over the BPE contract when it is assigned.

The second statement follows from point (iii) of Proposition 13.

Proof of Proposition 15

The deadweight loss under the standard and APE contracts is equal to $\sum_{ts}(w^*_{ts} - c^*_{ts})\gamma^H_{ts} = (w^*_{ua} - c^*_{ua})\gamma^H_{ua}$ and $\sum_{ts}(w^\dagger_{ts} - c^\dagger_{ts})\gamma^H_{ts} = (w^\dagger_{ua} - c^\dagger_{ua})\gamma^H_{ua}$, respectively. Since $c^*_{ua} = c^\dagger_{ua} = 0$, the deadweight loss is smaller under the APE contract if

$$
\begin{gathered}
w^*_{ua} > w^\dagger_{ua} \\
\frac{c^*_{aa}}{P_{aa}} > c^\dagger_{aa} \\
\frac{\Delta V}{\Delta\Gamma_a}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right) \\
1 > P_{aa}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a} \\
\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a) > P_{aa}\tilde{P}_{au}\Gamma^H_a + P_{aa}\tilde{P}_{uu}\Gamma^H_u \\
(1 - P_{aa})\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}\Gamma^H_a - P_{aa}\tilde{P}_{uu}\Gamma^H_u > 0 \\
P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\left[\tilde{P}_{aa} - \Gamma^H_a - P_{aa}(1 - \Gamma^H_a)\right] > 0 \\
P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(P_{aa} + b_a - \Gamma^H_a - P_{aa} + P_{aa}\Gamma^H_a) > 0 \\
P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\left[b_a - (1 - P_{aa})\Gamma^H_a\right] > 0 \\
P_{au}(P_{au} - b_a)\Gamma^H_a + \tilde{P}_{uu}(b_a - P_{au}\Gamma^H_a) > 0,
\end{gathered}
$$

which is always true since in the APE contract we have $b_a \in (P_{au}\Gamma^H_a, P_{au}]$.

Similarly, the deadweight loss under the PED-DL contract is given by

$$\begin{aligned}
&\sum_{ts} \left(\hat{w}_{ts} - \hat{c}_{ts}\right) \gamma_{ts}^{H} \\
&\quad = (\hat{w}_{ua} - \hat{c}_{ua})\gamma_{ua}^{H} \\
&\quad = \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^{H}}{\tilde{\gamma}_{ua}^{H}}\right)\hat{c}_{au} \\
&\quad = \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^{H}}{\tilde{\gamma}_{ua}^{H}}\right)\frac{\Delta V}{\Delta\Gamma_{A}}\frac{\Gamma_{u}^{H}}{\tilde{P}_{au} - \Gamma_{A}^{H}} \\
&\quad = \left(\frac{P_{au}\tilde{\gamma}_{ua}^{H} - P_{aa}\tilde{\gamma}_{aa}^{H}}{P_{aa}\tilde{P}_{ua}\Gamma_{u}^{H}}\right)\frac{\Delta V}{\Delta\Gamma_{A}}\frac{\Gamma_{u}^{H}}{\tilde{P}_{au} - \Gamma_{A}^{H}} \\
&\quad = \frac{\Delta V}{\Delta\Gamma_{A}}\frac{P_{au}\tilde{\gamma}_{ua}^{H} - P_{aa}\tilde{\gamma}_{aa}^{H}}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_{A}^{H}\right)}.
\end{aligned}$$

To see that the deadweight loss in a PED-DL contract is always lower than that in a BPE contract when the PED-DL one is optimal we calculate

$$\begin{aligned}
&\frac{\Delta V}{\Delta\Gamma_{a}}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta\Gamma_{A}}\frac{P_{au}\tilde{\gamma}_{ua}^{H} - P_{aa}\tilde{\gamma}_{aa}^{H}}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_{A}^{H}\right)} \\
&1 > \frac{P_{au}\tilde{\gamma}_{ua}^{H} - P_{aa}\tilde{\gamma}_{aa}^{H}}{\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_{A}^{H}\right)} \\
&P_{au}\tilde{\gamma}_{ua}^{H} - P_{aa}\tilde{\gamma}_{aa}^{H} - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma_{A}^{H} < 0 \\
&P_{au}\tilde{P}_{ua}\Gamma_{u}^{H} - P_{aa}\tilde{P}_{aa}\Gamma_{a}^{H} - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma_{A}^{H} < 0 \\
&P_{au}P_{ua}\Gamma_{u}^{H} - P_{aa}P_{aa}\Gamma_{a}^{H} - P_{ua}P_{au} + P_{ua}\Gamma_{A}^{H} + b_{u}P_{au}\Gamma_{u}^{H} - b_{a}P_{aa}\Gamma_{a}^{H} - b_{u}P_{au} + b_{a}P_{ua} + b_{u}b_{a} + b_{u}\Gamma_{a}^{H} < 0 \\
&P_{au}P_{ua}\underbrace{(\Gamma_{u}^{H} - 1)}_{-\Gamma_{a}^{H}} - P_{aa}P_{aa}\Gamma_{a}^{H} + P_{ua}\Gamma_{A}^{H} + b_{u}(P_{au}\Gamma_{u}^{H} - P_{au} + \Gamma_{a}^{H}) + b_{a}(P_{ua} + b_{u} - P_{aa}\Gamma_{a}^{H}) < 0 \\
&P_{ua}\Gamma_{a}^{H}\underbrace{(1 - P_{au})}_{P_{aa}} - P_{aa}P_{aa}\Gamma_{a}^{H} + b_{u}(P_{au}(\Gamma_{u}^{H} - 1) + \Gamma_{a}^{H}) + b_{a}(\tilde{P}_{ua} - P_{aa}\Gamma_{a}^{H}) < 0 \\
&P_{aa}\Gamma_{a}^{H}(P_{ua} - P_{aa}) + b_{u}\Gamma_{a}^{H}(1 - P_{au}) + b_{a}(\tilde{P}_{ua} - P_{aa}\Gamma_{a}^{H}) < 0 \\
&P_{aa}\Gamma_{a}^{H}(P_{ua} - P_{aa} + b_{u}) + b_{a}(\tilde{P}_{ua} - P_{aa}\Gamma_{a}^{H}) < 0 \\
&P_{aa}\Gamma_{a}^{H}(\tilde{P}_{ua} - P_{aa}) + b_{a}(\tilde{P}_{ua} - P_{aa}\Gamma_{a}^{H}) < 0.
\end{aligned}$$

Recall that for the PED-DL contract to be optimal $b_a \in \left[-P_{aa}, -P_{aa}\Gamma_a^H\right]$. Since the above inequality is linear in b_a, but its effect on the LHS is not straightforward, we can check that it holds at the extremes of the interval. At $b_a = -P_{aa}$ we have

$$-P_{aa}\tilde{P}_{ua} + P_{aa}^{2}\Gamma_{a}^{H} + P_{aa}\tilde{P}_{ua}\Gamma_{a}^{H} - P_{aa}^{2}\Gamma_{a}^{H} = -P_{aa}\tilde{P}_{ua} + P_{aa}\tilde{P}_{ua}\Gamma_{a}^{H} = P_{aa}\tilde{P}_{ua}(\Gamma_{a}^{H} - 1) < 0.$$

At $b_a = -P_{aa}\Gamma_a^H$ we have

$$-P_{aa}\tilde{P}_{ua}\Gamma_a^H + P_{aa}^2(\Gamma_a^H)^2 + P_{aa}\tilde{P}_{ua}\Gamma_a^H - P_{aa}^2\Gamma_a^H = P_{aa}^2(\Gamma_a^H)^2 - P_{aa}^2\Gamma_a^H = P_{aa}^2\Gamma_a^H(\Gamma_a^H - 1) < 0.$$

This proves that the PED-DL contract always features a smaller deadweight loss than the BPE contract.